

Multi-Objective Optimization for Inventory Control In Serial-Production Systems with Dependent Demand and Uncertainty of Lead Times

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Abstract: Supply planning for multi-period serial production systems under lead-time uncertainties is considered in this paper. The order policy is periodic order quantity (POQ) and objective is to find the planned lead-time and periodicity for the total items in order to minimize the expected fixed ordering, holding and partial backlogging costs and to maximize the customer service level for the finished product. In this paper assumed a percentage of items at each stage are scraps. For this new problem, a new mathematical model is suggested and then finding the optimal solution for this model.

Keywords: Serial production systems; planned lead time; Periodic order quantity; Uncertainty; Supply chain, partial backlogging

1. Introduction

In an industrial context, data are often imprecise or uncertain. In production management, it may for instance be the case for the demand, the lead times, the resources required, their capacities, the transportation times, the inventory or production costs, etc. When analyzing the state of the art on this subject, it can be seen that the uncertainty on the demand is a great focus of the literature, while the uncertainty on costs and capacities is also often considered.

By optimizing items supplies enterprises can save money and increase the customer satisfaction. In the literature of production planning and inventory control, assumed that lead-time is equal to zero or constant.

In reality, lead-times are rarely constant; unpredictable events can cause random delays. For different reasons (machine breakdowns, transport delays, or quality problems, etc.), the item lead times (time of item delivery from an external supplier or processing time for the semi-finished product at the previous level) are often random. To reduce the effective of random factors, firms implement safety lead times, but safety lead times are expensive. In contrast, if the safety lead times are not enough, we have stock-out and then backlogging cost.

There are various papers, which consider to uncertainty lead time, but most of them used lot-for-lot ordering policy and ignore the customer's service level, and in this paper, is assumed percentage of the lack is

backlogs and the percentage remaining is the lost sells.

The aim of this paper is determining planning lead-times, ordering quantity and periodic time which sum of setup cost, holding cost of all items, holding cost of finished product and backlogging costs minimized. There are several publications one level with multi items assembly system. Researcher using difference method for finding optimal solution such as minimize sum of total costs. In [1] consider to this model by using lot-for-lot order police. The objective is to determine safety lead-time such as minimize sum of holding costs. In [2] used Markovian model for a dynamical multi-period planning and one level assembly system and the aim was finding safety lead time such as minimize the sum of holding and backlogging costs and [3] were using simulated annealing for finding safety lead time. In [4] consider to one level assemble system again and used a Branch and Bound method for finding optimal planned lead time with lot-for-lot order police. The aim was to find the optimal MRP offsetting such as minimize the sum of the setup cost and average holding costs for the items, while satisfying a desired service level. The recent research for this problem is doing by [5] which consider to a multi-period serial production system when lead times for all items are uncertainties but don't consider to partial back-order and customer service level.

2. Notation and assumptions

In this research we consider multi-stage Planning supply items with dependent demand with uncertainty lead-time and partial back-order with POQ order policy.

The assumption of this model is as follow:

- A percentage of items at each stage are west
- The demand at each period is constant
- Ordering policy is POQ
- Lead time for each stage is uncertainty

- Shortages is allow and it is with partial backlogging

The raw materials are released at stage N, the semi-finished products are processed at stage n_1, n_2, \dots and finally, the finished product is produced.

The objective is to find the item planned lead time at each stage and periodicity (P) for minimizing the sum of the holding costs for the items of each level and back-ordering cost, lost sale cost, holding cost for the finished product and maximize customer service level.

I. Notation

- t : Index of period's $t=1, 2, 3,$
- A : ordering cost per order,
- D : Demand for finished product
- P : Time periods for each ordering
- h : Per unit holding cost per time unit
- $\hat{\pi}$: Backlogging costs for each stage in each period
- π : Lost Sales for each item in each period
- s : Maximum unit Backordering
- b : Maximum unit backlogging
- β : Percentage of Backordering which is Backlogging
- α_i : present of west for stage i.
- l_i : Random lead time for stage i.
- $1 - \varepsilon$: Service level
- $l = \sum_{i=1}^N l_i$: Total lead-time of the system.
- x_i : Planned lead time for stage i.
- $x = \sum_{i=1}^N x_i$: Sum of planned lead-time.
- $f(l_i)$: Distribution of lead-time in stage i.
- $f(l)$: Convolution of lead-time.
- C_i : Production cost per item at stage i.

Variables

- P: periodicity
- X: planned lead time

3. Model development

The lead-time is assumed probabilistic. The planned lead-time is x_i for stage i. The orders

for products are made at the beginning of the periods 1, p+1, 2p+1... and there is no order made in other periods. Order quantity is constant and equal to PD (P is a decision variable)

The objective in this system is to find the planned lead-time and periodicity for the total items in order to minimize the expected costs and maximize the customer service. Costs are holding costs for the all items of each stage and back-ordering cost, lost sale cost, holding cost for finished product and production/assemble cost.

3.1. The system costs

I. Production/Assemble costs

In this system, at each stage, a percentage of items are scrap therefore according to the Fig.1 the total number of item in each stage is equal to:

$$Q_i = \frac{p \times D}{\prod_{j=i}^m (1 - \alpha_j)} \tag{1}$$

Then production/assemble costs is

$$C_1(x, p) = \sum_{i=1}^m C_i \times Q_i = \sum_{i=1}^m C_i \times \frac{p \times D}{\prod_{j=i}^m (1 - \alpha_j)} = p \times D \times \sum_{i=1}^m \frac{C_i}{\prod_{j=i}^m (1 - \alpha_j)} \tag{2}$$

II. Final product costs

Because of uncertainly lead-time, there are three states in action. One of them is planned lead time equal to actual lead time (Fig. 2) and the second state is planned lead-time is smaller than actual lead-time (Fig. 3) and the third state is planned lead time is bigger than the actual lead time (Fig. 4)

The cost for these stats are as follow:

State 1:

This state has not back-order cost and the costs for this stat is equal to:

$$C_2(x, p) = [A + (p-1)hD + (p-2)hD + \dots + 2hD + hD] \times f(l=x) \tag{3}$$

$$= \left[A + \frac{p(p-1)}{2} hD \right] \times f(l=x)$$

Where $f_t(l)$ is the convolution of lead-time.

State 2:

If at a stage, a job is not received as planned, a delay is incurred at delivery of production and it is maybe incurred back-ordering at finished product. In this case, it is assumed percentage of the lack is backlogs and the percentage remaining is the lost sells.

$$C_3(x, p) = \left[A + \frac{(p-(l-x))(p-1-(l-x))}{2} hD + \hat{\pi} \times \beta \times D \frac{(l-x)(l-x+1)}{2} + \pi \times (1-\beta) \times \frac{(l-x)(l-x+1)}{2} \right] \times P(l > x) \tag{4}$$

State 3:

$$C_4(x, p) = \left[A + \frac{p(p-1)}{2} hD + hpD(x-l) \right] \times P(l < x) \tag{5}$$

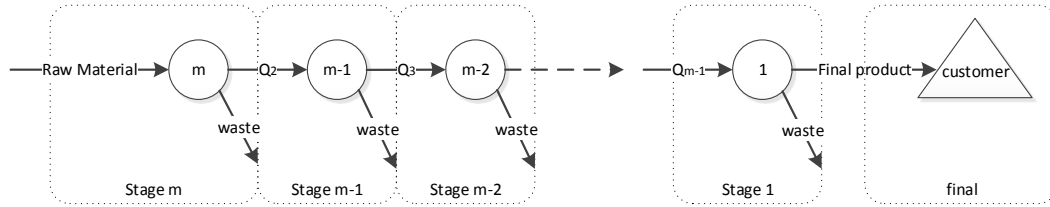


Fig. 1: The m-stage manufacturing system

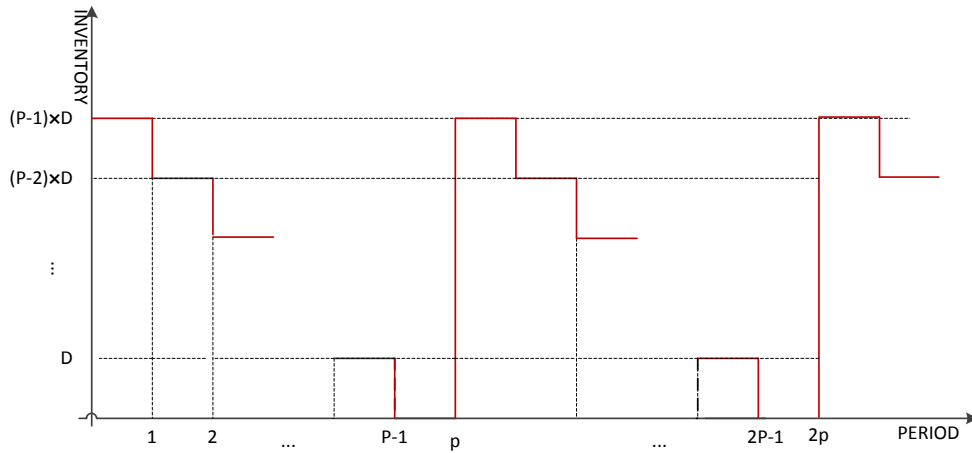


Fig. 2: planned lead time equal to actual lead time

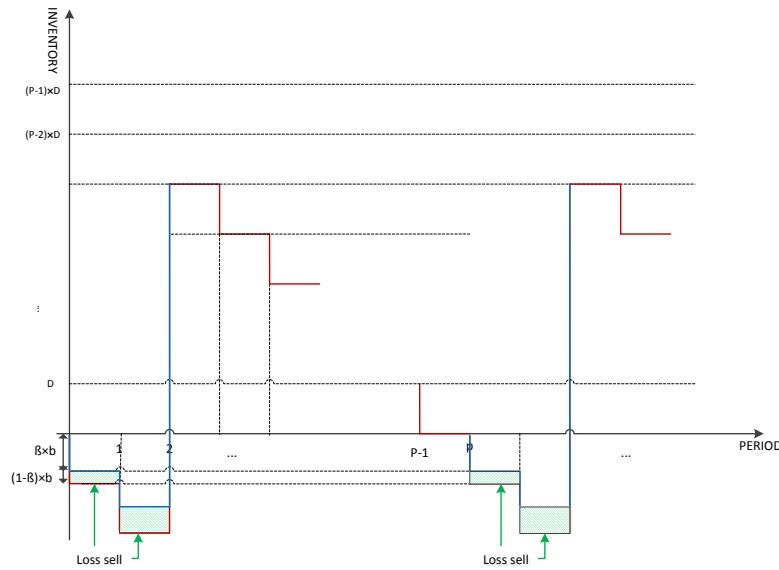


Fig. 3: An illustration of the planning problem when planned lead-time is smaller than actual lead-time

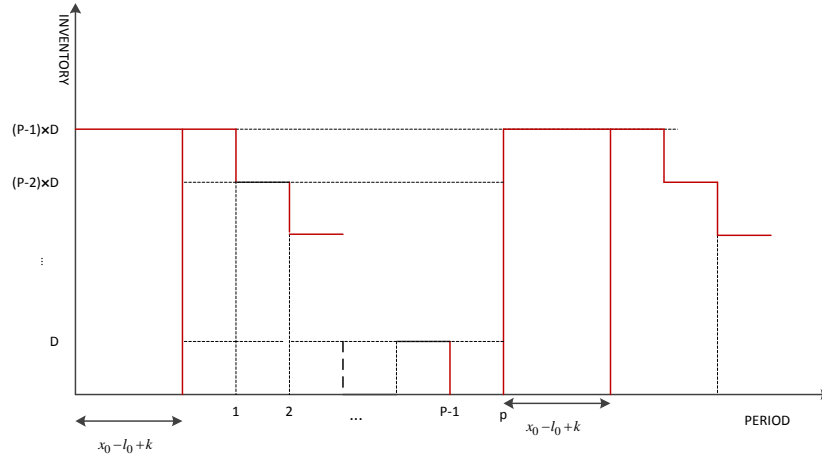


Fig. 4: An illustration of the planning problem when planned lead-time is bigger than the actual lead-time

With summarize the Eq(2), Eq(3), Eq(4) and Eq(5), have:

$$\begin{aligned}
 C(x, p) &= C_1(x, p) + C_2(x, p) + C_3(x, p) + C_4(x, p) = \\
 & p \times D \times \sum_{i=1}^m \frac{C_i}{\prod_{j=i}^m (1-\alpha_j)} + A + \frac{p(p-1)}{2} hD + \\
 & (hpD(x-l)) \times P(l < x) - \frac{hD(l-x)(2p-1)}{2} \times P(l > x) \quad (6) \\
 & + \left(\hat{\pi} \times \beta \times D \frac{(l-x)(l-x+1)}{2} \right) \times P(l > x) \\
 & + \left(\pi \times (1-\beta) \times D \frac{(l-x)(l-x+1)}{2} \right) \times P(l > x)
 \end{aligned}$$

Explicit forms of the total cost, $\hat{C}(x, p)$ has been presented as follow:

$$\begin{aligned}
 \hat{C}(x, p) &= \frac{C(x, p)}{p} = \\
 & D \times \sum_{i=1}^m \frac{C_i}{\prod_{j=i}^m (1-\alpha_j)} + \frac{A}{p} + \frac{(p-1)}{2} hD + \\
 & (hD(x-l)) \times P(l < x) - \frac{hD(l-x)(2p-1)}{2p} \times P(l > x) \quad (7) \\
 & + \left(\hat{\pi} \times \beta \times D \frac{(l-x)(l-x+1)}{2p} \right) \times P(l > x) \\
 & + \left(\pi \times (1-\beta) \times D \frac{(l-x)(l-x+1)}{2p} \right) \times P(l > x)
 \end{aligned}$$

An explicit form for the stock-out probability $sp(x, l)$ is the following:

$$sp(x, l) = p(l > x) \quad (8)$$

$$\text{Where } x = \sum_{i=1}^N x_i \text{ and } l = \sum_{i=1}^N l_i .$$

The problem is to minimize the total costs given by (7) while maximizing the service level, i.e., minimizing the stock out probability given by (8)

- Minimize $\hat{C}(x, p)$
- Minimize $sp(x, l)$

For solving the model we should determine the distribution of $l = \sum_{i=1}^m l_i$. We assume that lead-times' distribution are independent. Therefore distribution of $l = \sum_{i=1}^m l_i$ will be calculated simply by using convolution concept. For more details, please refer to [5].

3.2. Optimal solution properties

The final model can be written as follows:

$$\begin{aligned}
 \text{Min } \hat{C}(x, p) = & \\
 \left(\begin{aligned}
 & D \times \sum_{i=1}^m \frac{C_i}{\prod_{j=i}^m (1-\alpha_j)} + \frac{A}{p} + \frac{(p-1)hD}{2} + \\
 & (hD(x-l)) \times P(l < x) - \frac{hD(l-x)(2p-1)}{2p} \times P(l > x) \\
 & + \left(\hat{\pi} \times \beta \times D \frac{(l-x)(l-x+1)}{2p} \right) \times P(l > x) + \\
 & \left(\pi \times (1-\beta) \times D \frac{(l-x)(l-x+1)}{2p} \right) \times P(l > x)
 \end{aligned} \right) \quad (9)
 \end{aligned}$$

Subject to

$$p(l > x) \leq \varepsilon \quad (10)$$

Where $x = \sum_{i=1}^N x_i$ and $l = \sum_{i=1}^N l_i$.

• Solving the model

The function of $\hat{C}(x, p)$ is convex. Therefore we can find the optimal solution for it.

By replacing this solution on the constraint, if $p(l > x)$ smaller or equal to ε , then this answer is optimal solution, but if $p(l > x) \leq \varepsilon$ is not true, then we replace ε with $p(l > x) \leq \varepsilon$ on $\hat{C}(x, p)$ and find the optimal solution.

Theorem 1: The objective function $\hat{C}(x, p)$ in Eq. (9) is convex.

$$\begin{aligned}
 \frac{\partial \hat{C}(x, p)}{\partial x} = & \\
 & -\frac{D}{p} \times (h + \hat{\pi} \times \beta + \pi \times (1-\beta)) \times \int_{l \geq x} (l-x) f(l) dl \\
 & -\frac{D}{2p} \times h \times \int_{l \geq x} f(l) dl - \frac{D}{2p} \times (h + \hat{\pi} \times \beta + \pi \times (1-\beta)) \times \int_{l \geq x} f(l) dl \\
 \frac{\partial^2 \hat{C}(x, p)}{\partial x^2} = & \frac{D}{p} (h + \hat{\pi} \times \beta + \pi \times (1-\beta)) \times \int_{l \geq x} f(l) dl \quad (11)
 \end{aligned}$$

Eq (11) is positive then, objective function is strictly convex.

The total cost equation $\hat{C}(x, p)$ is convex therefore for finding the optimal solution used $\frac{\partial \hat{C}(x, p)}{\partial x} = 0$.

$$\begin{aligned}
 \frac{\partial \hat{C}(x, p)}{\partial x} = 0 \Rightarrow & \\
 \int_{l \geq x} (l-x+0.5) f(l) dl = & \frac{p \times h}{(h + \hat{\pi} \times \beta + \pi \times (1-\beta))} \quad (12)
 \end{aligned}$$

And if the Distribution of lead-time in stage i is discrete, we can use

$$\begin{aligned}
 \Delta \hat{C}(x, p) = \hat{C}(x, p) - \hat{C}(x-1, p) \geq 0 \text{ and} \\
 \Delta \hat{C}(x, p) = \hat{C}(x+1, p) - \hat{C}(x, p) \leq 0 \text{ for optimal} \\
 \text{planned lead-time.}
 \end{aligned}$$

By used $\Delta \hat{C}(x, p) = \hat{C}(x, p) - \hat{C}(x-1, p) \geq 0$, we have:

$$\begin{aligned}
 \Delta(E[\hat{C}(x, p)]) = & \\
 h \times D + \frac{D}{2p} (h + \hat{\pi} \times \beta + & \\
 \pi \times (1-\beta)) \times \sum_{l > x} (2 \times (l-x) - 1) p(L=l) & \\
 + \frac{D}{2p} (h + \hat{\pi} \times \beta + \pi \times (1-\beta)) \times \sum_{l > x} p(L=l) & \\
 \Rightarrow \Delta(E[\hat{C}(x, p)]) = h \times D & \\
 + \frac{D}{p} (h + \hat{\pi} \times \beta + \pi \times (1-\beta)) \times \sum_{l > x} ((l-x) - 1) p(L=l) & \\
 \Delta(E[\hat{C}(x, p)]) \geq 0 \Rightarrow & \\
 h \times D - \frac{D}{p} (h + \hat{\pi} \times \beta + \pi \times (1-\beta)) \times \sum_{l > x} ((l-x) - 1) p(L=l) \geq 0 & \\
 \Rightarrow \sum_{l > x} (l-x-1) p(L=l) \leq \frac{h \times p}{h + \hat{\pi} \times \beta + \pi \times (1-\beta)} \quad (13)
 \end{aligned}$$

By simplify Eq(13) we have:

$$\begin{aligned}
 \sum_{l > x} (l-x-1) p(L=l) = & \\
 p(L=x+2) + 2 \times p(L=x+3) + 3 \times p(L=x+4) + \dots = & \\
 (p(L=x+2) + p(L=x+3) + p(L=x+4) + \dots) & \\
 + (p(L=x+3) + p(L=x+4) + p(L=x+5) + \dots) + \dots & \\
 = 1 - F(x+1) + 1 - F(x+2) + \dots = \sum_{l > x} (1 - F(l)) & \\
 \Rightarrow \sum_{l > x} (l-x-1) p(L=l) = \sum_{l > x} (1 - F(l)) & \\
 \Rightarrow \sum_{l > x} (1 - F(l)) < \frac{hp}{h + \hat{\pi} \times \beta + \pi \times (1-\beta)} \quad (14)
 \end{aligned}$$

For finding periodic order quantity parameter (p), we respectively replace the values 1, 2, ...

instead of p at the cost function and, obtain total cost by Eq. (9) and optimal value of x .

Noted that l is actual lead-time for one period, but we consider to multi period modelling, then planned lead time (x) is defined for $p \times D$ items. Therefore planned lead time for one period is equal to $\frac{x p}{p}$ which x^p is planned lead-time for p period.

4. Numerical example

Consider five stages Planning supply items with dependent demand with following parameters

$$m = 5, A = 100, h = 10, \hat{\pi} = 10, \pi = 5, D = 1, \beta = 0.8, C_i = 20, \varepsilon = 0.05, \text{ and } \alpha_i = 0.01$$

Distribution of lead-time at each stage is as follow:

Table 1: distribution of lead-time at each stage

l	1	2	3	4	5
$p(L_1 = l)$	0.50	0.30	0.10	0.05	0.05
$p(L_2 = l)$	0.2	0.2	0.3	0.1	0.2
$p(L_3 = l)$	0.15	0.3	0.2	0.15	0.2
$p(L_4 = l)$	0.4	0.1	0.2	0.15	0.15
$p(L_5 = l)$	0.1	0.2	0.3	0.15	0.25

Solution:

According to Eq. (14) have:

$$\sum_{l > x} (1 - F(l)) < \frac{hp}{h + \hat{\pi} \times \beta + \pi \times (1 - \beta)} \Rightarrow \sum_{l > x} (1 - F(l)) < \frac{10 \times p}{20}$$

According to Distribution of lead-times, the minimum of $l = \sum_{i=1}^N l_i$ is equal to 5 and the maximum of it equal to 25. Therefore, convolution of lead-time with discrete distribution is as follow:

$$\begin{aligned} F(5) &= 0.000600 & F(6) &= 0.0041100 & F(7) &= 0.0154700 \\ F(8) &= 0.041117 & F(9) &= 0.088155 & F(10) &= 0.1613575 \\ F(11) &= 0.260955 & F(12) &= 0.380565 & F(13) &= 0.5100080 \\ F(14) &= 0.63467 & F(15) &= 0.748945 & F(16) &= 0.8389131 \\ F(17) &= 0.9053506 & F(18) &= 0.9491219 & F(19) &= 0.9753488 \\ F(20) &= 0.9892532 & F(21) &= 0.99598695 & F(22) &= 0.9986732 \\ F(23) &= 0.999637575 & F(24) &= 0.999926325 & F(25) &= 1 \end{aligned}$$

Step 1: p=1;

$$\sum_{l > x} (1 - F(l)) < \frac{10 \times p}{20} \Rightarrow \sum_{l > x} (1 - F(l)) < \frac{10}{20} \Rightarrow x^* = 16$$

$$\hat{C}(16,1) = 254.4848$$

Step 2: p=2;

$$\sum_{l > x} (1 - F(l)) < \frac{10 \times p}{20} \Rightarrow \sum_{l > x} (1 - F(l)) = 1 \Rightarrow x^* = 14$$

$$\hat{C}(14,2) = 189.0494$$

Step 3: p=3;

$$\sum_{l > x} (1 - F(l)) < \frac{10 \times p}{20} \Rightarrow \sum_{l > x} (1 - F(l)) = 1.5 \Rightarrow x^* = 13$$

$$\hat{C}(13,3) = 176.0671$$

Step 4: p=4;

$$\sum_{l > x} (1 - F(l)) < \frac{10 \times p}{20} \Rightarrow \sum_{l > x} (1 - F(l)) = 2 \Rightarrow x^* = 13$$

$$\hat{C}(13,4) = 172.6992$$

Step 5: p=5;

$$\sum_{l > x} (1 - F(l)) < \frac{10 \times p}{20} \Rightarrow \sum_{l > x} (1 - F(l)) = 2.5 \Rightarrow x^* = 12$$

$$\hat{C}(12,5) = 168.668$$

Step 6: p=6;

$$\sum_{l > x} (1 - F(l)) < \frac{10 \times p}{20} \Rightarrow \sum_{l > x} (1 - F(l)) = 3 \Rightarrow x^* = 11$$

$$\hat{C}(11,6) = 167.5983$$

Step 6: p=7;

$$\sum_{l > x} (1 - F(l)) < \frac{10 \times p}{20} \Rightarrow \sum_{l > x} (1 - F(l)) = 3.5 \Rightarrow x^* = 11$$

$$\hat{C}(11,6) = 170.2528$$

Cost reduce form $p=1$ until $p=5$, and then cost is increased, Therefore, optimal solution without considering to customer service level, is equal to $x^* = 12, p^* = 5, x_p^* = x \times p = 12 \times 5 = 60$

$$\hat{C}^*(12,5) = 55.21071$$

And the customer's service level is equal to:

$$p(l > x) = 1 - F(x) = 1 - F(12) = 1 - 0.380565$$

Therefore, if customer's service level is smaller than 0.38, this answer is the optimal, but if customer's service level is bigger than 0.38, then we replace ε with $p(l > x) \leq \varepsilon$ on $\hat{C}(x, p)$ and finding the optimal solution. Table 2 represent the optimal solution for various customer service level.

Table 2: The optimal solution for various customer service level.

Service level ($1-\varepsilon$)	Optimal planned lead time (x^*)	p	stock-out probability	Total cost
20%	12	6	0.619435	167.5983
30%	12	6	0.619435	167.5983
38%	12	6	0.619435	167.5983
40%	13	5	0.489992	172.6784
50%	13	5	0.489992	172.6784
60%	14	5	0.36533	177.9776
70%	15	5	0.251055	184.4968
80%	16	5	0.1610869	192.1175
85%	17	5	0.0946494	200.5982
90%	17	5	0.0946494	200.5982
95%	19	5	0.0246512	219.2327
98%	20	5	0.0107468	229.0026
100%	25	5	0	278.8778

5. Conclusion

In this paper we consider to multi-stage planning supply items with dependent demand with stochastic item's lead-times. We use new method for modeling this model. The main goal in this model is minimize sum of inventory holding cost, back-order cost and setup cost in order to find the optimize solution of item planned lead times and order periodicity for assembly systems. A MRP approach with periodic order quantity (POQ) policy is used for the supply planning of items. We consider a new approach for modeling multi-stage Planning supply items with dependent demand under uncertainty item's lead-times. We proved that the cost function is convex and then using the convex concepts for finding optimal solutions. By using of this equation we can find the optimal solution for this model.

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