Three Jet Rate and Thrust Distribution at NNLO

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ABSTRACT: We have studied hadronic events from e^+e^- annihilation data at different centre-of-mass energies .The operation of the AMY and OPAL collaborations offers a unique opportunity to test QCD by considering different observables. The main results concern with the measurement of the strong coupling constant, α_s, from hadronic event shapes. By using the three jet rates and thrust distributions, we extract the strong coupling constant, α_s, at Next to Next Leading Order (NNLO) from AMY and OPAL data. The results are consistent with the running of α_s, expected from QCD predictions.

Keywords: Three-jet rate; NNLO; QCD; Strong coupling constant

1. Introduction

In recent decades some papers reported on the first calculations of NNLO corrections to event shape variables and three jet observable, and discussed their phenomenological impact,[1,2].

Three-jet production at tree-level is induced by the decay of a virtual photon (or other neutral gauge boson) into a quark-antiquark-gluon final state. At higher orders, this process receives corrections from extra real or virtual particles. The individual partonic channels that contribute through to NNLO are cited in ref [1]. For a given partonic final state, the event shape observable "y" is computed according to the same definition as in the experiment, which is applied to partons instead of hadrons. At leading order, all three final state partons must be well separated from each other, such that "y" differs from the trivial two-parton limit. At NLO, up to four partons can be present in the final state, two of which can be clustered together, whereas at NNLO, the final state can consist of up to five partons, and as many as three partons can be clustered together. The more partons in the final state, the better one expect the matching between theory and experiment to be [3].

Data from e^+e^- annihilation provide us with one of the cleanest ways of probing our quantitative understanding of QCD and hence giving us an opportunity to measure the strong coupling between quarks and gluons. Three-jet production cross sections and related event shape distributions in e^+e^- annihilation processes are classic hadronic observables which can be measured very accurately and provide an ideal proving ground for testing our understanding of strong interactions.

Jet observables in e^+e^- annihilation can be used to extract the value of the strong coupling constant, α_s, [4-6]. This applies in particular to three-jet observables, where the leading-order parton process is proportional to α_s. In order to extract the numerical value from the AMY and OPAL data, precise theoretical calculations are necessary, calling for a next-to next-to-leading order (NNLO) calculation.

In this paper by studying the dependence of the relative three jet rates in different energies to their relative α_s, NNLO in different experiments will be tested. Then we will measure the coupling constant by using three jet observable at NNLO from AMY and OPAL data. The reason for doing such measurement is that in refs [7,8]; the strong coupling, α_s, has been determined from the measured moments of particle momenta within three jet rates and thrust distribution using NNLO. One might anticipate a priority that this would yield a less precise determination of α_s than the differential distribution of jet rate, because the theory lacks resumption of large logarithms, and the moments include regions of phase space where hadronization effects are large [9]. Nevertheless, the comparison of α_s determined in this way with that obtained from the jet and thrust distribution should provide an illumination test of the adequacy of QCD in this area.

2. JADE algorithm

We separate two and three jet events by employing the jet clustering algorithm introduced by the JADE group [10]. In this algorithm the scaled mass spread defined as $Y_{ij} = \frac{m_{ij}^2}{E_{vis}^2}$ with $m_{ij}^2 = 2E_iE_j(1-\cos\theta_{ij})$ is calculated for each pair of particles in the event. If the smallest of the Y_{ij} values is less than a parameter Y_{cut}, the corresponding pair of particles is combined into a cluster by summing the four momenta. This process is repeated, using all combinations of clusters and remaining particles, until all the Y_{ij} values exceed Y_{cut}. The clusters remaining at this stage are defined as the jets.

The distribution of jet multiplicities obtained by these clustering algorithms depends on the jet defining parameter Y_{cut}. For small Y_{cut}, many jets are found because of the hadronization of fluctuation process, whereas for large Y_{cut}, mostly two jet events are found and the $q\bar{q}g$ -events are not resolved. However, Monte Carlo studies show that there is a range of cluster parameters, for which QCD effects can be resolved and the fragmentation effects are sufficiently small. In the

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following, the parameter Y_{cut}=0.04 is used which is found to be a reasonable cut [10].

In Fig.(1), we show three jet fraction for different Y_{cut}. The decrease of the 3-jet rate at large Y_{cut} is clearly visible. Our results are completely consistent with the results obtained by the JADE scheme [11,12] .

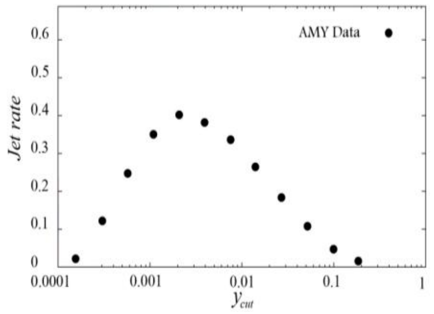

Fig.1. three jet fraction for different Y_{cut}s

3. The evidence for using of NNLO

So far the three-jet rate and related event shapes have been calculated [13] up to the next-to-leading order (NLO), improved by a resumption of leading and subleading infrared logarithms and by the inclusion of power corrections. QCD studies of event shape observables at LEP [14] are based around the use of NLO parton-level event generator programs [15]. As expected, the current error on α_s from these observables [16] is dominated by the theoretical uncertainty. Clearly, to improve the determination of α_s, the calculation of the NNLO corrections to these observables becomes mandatory. We present here comparing the values of strong coupling constant at LO, NLO with QCD predictions for considering the importance of α_s calculation at NNLO. For this reason we use of coefficients of the leading-order for various values of Y_{cut} in table (1) in ref [17] and the data in ref [18] for deriving strong coupling constant at LO and NLO.

Table 1. The values of strong coupling constant in 133 GeV at LO

Y_{cut}	0.1	0.06	0.03	0.01	0.001
LO	0.2096	0.1660	0.1416	0.1352	0.0056

Table (1) shows that the value of α_s changes in different Y_{cut}s and we can't give the QCD prediction for it. That's because we regard only three level Partonic contribution ($\gamma^* \to q\bar{q}g$) to three-jet final states in LO and ignore other vertexes.

Table 2. The values of strong coupling constant in 133 GeV at NLO

Y_{cut}	0.1	0.06	0.03	0.01	0.001
NLO	0.1191	0.1267	0.1219	0.1479	0.1598

In table (2) α_s values are near to each other but NLO doesn't give QCD prediction exactly. In this order contributions of one loop ($\gamma^* \to q\bar{q}g$), three level ($\gamma^* \to q\bar{q}gg$) and three level ($\gamma^* \to q\bar{q}q\bar{q}$) vertexes are regarded but two loop ($\gamma^* \to q\bar{q}g$)**,** one loop ($\gamma^* \to q\bar{q}gg$) and one loop ($\gamma^* \to q\bar{q}q\bar{q}$) partonic contributions are ignored. Furthermore the five-parton contributions ($\gamma^* \to q\bar{q}ggg$, $\gamma^* \to q\bar{q}q\bar{q}g$) to three-jet-like final states at NNLO contain infrared real radiation singularities, which have to be extracted and combined with the infrared singularities [19] present in the virtual three-parton and four-parton contributions to yield a finite and exact result.

3. Three jet observables at NNLO

We report here on the calculation of NNLO corrections to the 3-jet cross section and related event shape variables. The knowledge of the NNLO corrections to the event shape distributions has important phenomenological impact on the extraction of strong coupling constant from AMY and OPAL data. The calculation of the α_s^3 corrections for three jet production is carried out using a developed parton-level event generator program JADE [10] which contains the relevant matrix elements with up to five external partons. Besides explicit infrared divergences from the loop integrals, the four-parton and five-parton contributions yield infrared divergent contributions if one or two of the final state partons become collinear or soft. In order to extract these infrared divergences and combine them with the virtual corrections, the antenna subtraction method [9] was extended to NNLO level [10] and related event-shape variables [11] into JADE.

The three-jet cross section is expanded as[13]

$$\frac{\sigma_{3-jet}}{\sigma_{tot}} = \frac{\alpha_s}{2\pi} A_{3-jet} + (\frac{\alpha_s}{2\pi})^2 B_{3-jet} + (\frac{\alpha_s}{2\pi})^3 C_{3-jet} \quad (1)$$

where σ_0 is the LO cross section for $e^+e^- \to hadrons$. The coefficients A_{3-jet}, B_{3-jet} and C_{3-jet} are calculated for $\mu^2 = Q^2$ [13].The measured three-jet cross-sections are shown in graphical form in Figure 2.

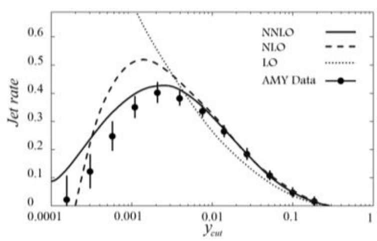

Fig 2: The scale variation of the three jet rate with the JADE jet algorithm

By comparing the three plots, we observe that the agreement for each of the jet rates becomes systematically better as the order of perturbation theory increases. With fitting our data with equation (1) the strong coupling constant α_s is derived. The values of α_s at LO, NLO, NNLO are presented in table 3.

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Table 3. α_s values for 60 GeV at LO, NLO and NNLO

	LO	NLO	NNLO
α_s	0.121±0.008.	0.123±0.005.	0.124±0.003

The values of strong coupling constant for OPAL data are calculated at NNLO in ref [20].By using this data [21] and NNLO model[13] We try to measure α_s at different energies in figure (3) and compare these values with each other and QCD prediction in figure (4) and table (4).

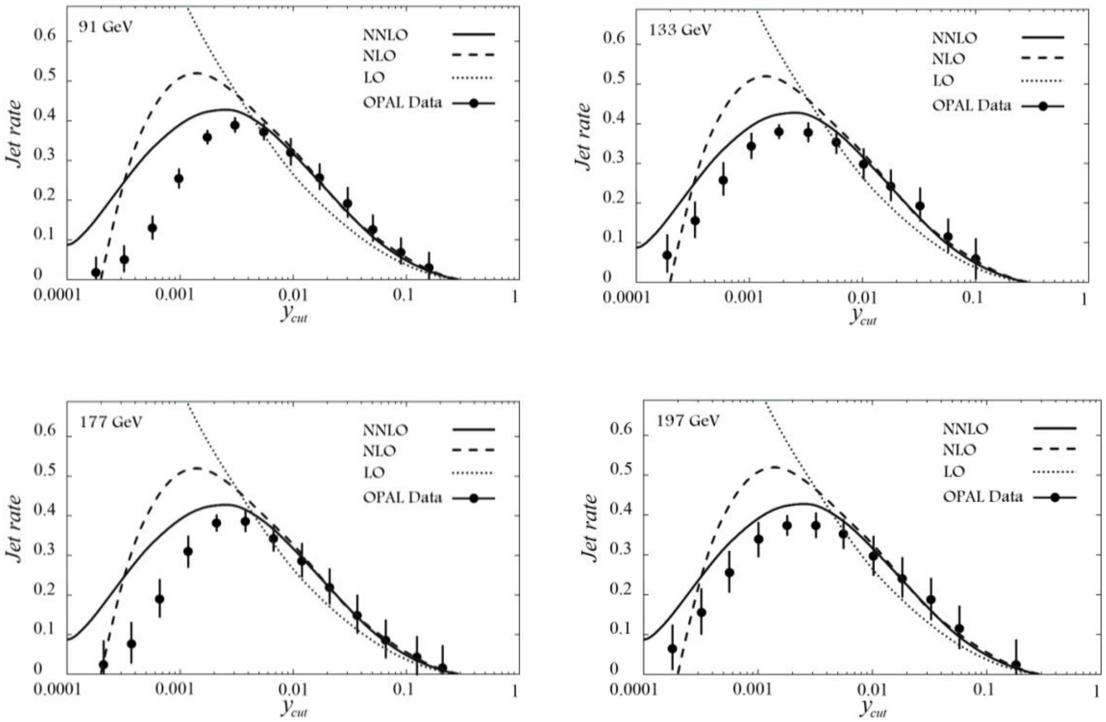

Fig 3: three jet fractions for different Y_{cut} [20,21]

Table 4. α_s values for different centre-of-mass energies from 60 to 197 GeV.

	60 GeV	91 GeV	133 GeV	177 GeV	197 GeV
α_s	0.124±0.003	0.119±0.006	0.112±0.005	0.104±0.004	0.105±0.007

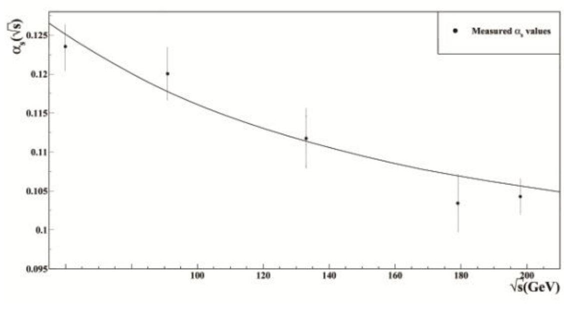

Fig 4. The running of α_s as a function of centre-of-mass energy.

Table 5. The values of strong coupling constant in 133GeV at NNLO

Y_{cut}	0.1	0.06	0.03	0.01	0.001
α_s	0.1171	0.1003	0.1100	0.1004	0.1610

Table (5) shows that as we expected the strong coupling at NNLO for different $Y_{cut}s$ is relatively constant. Of course we use of Delphi data in this table but we can test this result for other energies.

Table 6. The values of strong coupling constant in 194GeV at NNLO

Y_{cut}	0.1	0.06	0.03	0.01	0.001
α_s	0.1044	0.0957	0.0996	0.0966	0.1657

Table 7. The values of strong coupling constant in 182GeV at NNLO

Y_{cut}	0.1	0.06	0.03	0.01	0.001
α_s	0.0992	0.0929	0.0983	0.0930	0.1640

Table 8. The values of strong coupling constant in 91GeV at NNLO

Y_{cut}	0.1	0.06	0.03	0.01	0.001
α_s	0.1428	0.1327	0.1457	0.1142	0.1136

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5. Thrust, T

The global event-shape variable thrust, is defined as[22]:

$$T = \max\left(\frac{\sum \vec{p}_i.\vec{n}}{\sum |\vec{p}_i|}\right) \qquad (2)$$

where $\vec{p}_i$ is the momentum vector of particle i. the thrust axis $\vec{n}$ is the unit vector which maximizes the above expression. The value of the thrust can vary between 0.5 and 1.0. For an event with exactly two massive stable particles $T = \sqrt{Q^2 - 4m^2}/Q = 1 - \frac{2m^2}{Q^2}$ is the maximum allowed thrust. Note that for massless jet production the thrust (T) distribution is peaked close to T = 1 while for events containing a heavy quark pair it is peaked close to $T = \sqrt{Q^2 - 4m^2}/Q$. For example in 133 GeV (table (9)), 1-T is peaked between 0.01 to 0.02, because mass of bottom is 4.3 GeV and charm's mass is 1.5 GeV. Thus we conclude 1-T=0.0083±0.0020 in it's peak.

Table 9. thrust distribution in 133 GeV

1-T	0-.01	.01-.02	.02-.03	.03-.04	.04-.05	.05-.07	.07-.09	.09-.12	.12-.15	.15-.22	.22-.3
$\frac{1}{\sigma}\frac{d\sigma}{dT}$	4.37	20.4	20.4	10.5	6.7	4.7	3.64	1.68	1.36	1.09	.467

The perturbative expansion for the distribution of a generic observable T up to NNLO at e^+e^- centre-of-mass energy, $\sqrt{s}$, for a renormalisation scale μ^2 is given by [23]

$$\frac{1}{\sigma_{had}}\frac{d\sigma}{dT}(s,\mu^2,y) = \left(\frac{\alpha_s(\mu)}{2\pi}\frac{d\bar{A}}{dT} + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\right)\left(\frac{d\bar{B}}{dT} + \frac{d\bar{A}}{dT}\beta_0 \log\frac{\mu^2}{s}\right)$$
$$+\left(\frac{\alpha_s(\mu)}{2\pi}\right)^3\left(\frac{d\bar{C}}{dT} + 2\frac{d\bar{B}}{dT}\beta_0 \log\frac{\mu^2}{s} + \frac{d\bar{A}}{dT}(\beta_0^2 \log^2\frac{\mu^2}{s} + \beta_1 \log\frac{\mu^2}{s})\right) \qquad (3)$$
$$+O(\alpha_s^4)$$

in which

$$\beta_0 = \frac{11C_A - 4T_R N_F}{6}$$
$$\beta_1 = \frac{17C_A^2 - 10C_A T_R N_F - 6C_F T_R N_F}{6}, \qquad (4)$$

where the QCD colour factors are

$$C_A = N,\quad C_F = \frac{N^2-1}{2N},\quad T_R = \frac{1}{2} \qquad (5)$$

for N = 3 colours and N_F light quark flavours .The coefficients $\bar{A}$, $\bar{B}$ and $\bar{C}$ have been computed for several event-shape variables [24,25]. The measured normalized differential cross-sections T, for each of the energies are shown in graphical form in Figure 5. As the figure indicates, our real data are more consisted with NNLO compared to NLO or LO calculations, because it involves higher order terms in QCD calculations.

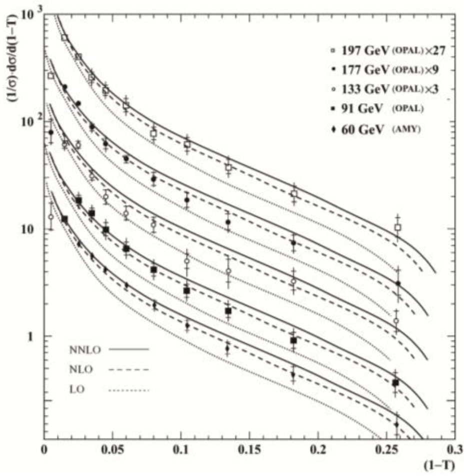

Fig 5. The thrust distribution at LO (dotted), NLO (dashed) and NNLO (solid) compared to experimental data from AMY and OPAL for Q =60 GeV to 197 GeV.

In all these results, the uncertainty due to the choice of the renormalization scale $x_\mu = \mu/\sqrt{s}$ yields a large contribution to the total error. Consequently, missing higher orders, whose effects on the values of the coupling are assessed by varying x_μ, are still important. The fitted values for α_s change considerably for different choices of the scale. The real value of α_s was estimated by comparing theory with data using the minimum of χ^2, such that every where χ^2 is minimum, it's corresponding α_s becomes our real strong coupling constant. The value of α_s was estimated by fitting the data. Separate fits were performed to each of the thrust observables at 60 GeV to 197 Centre of mass energies.(table(10)) in addition the values of strong coupling constant for different 1-T in 91-197 GeV are displayed in table (11).

Table 10. The values of strong coupling constant in 60-197 GeV at NNLO

	60 GeV	91 GeV	133 GeV	177 GeV	197 GeV
α_s	0.125±0.004	0.118±0.003	0.110±0.005	0.106±0.003	0.107±0.004

Table 11. The values of strong coupling constant for different 1-T in 60-197 GeV

1-T	**0.-.01**	**.01-.02**	**.02-.03**	**.03-.04**	**.04-.05**	**.05-.07**	**.07-.09**	**.09-.12**	**.12-.15**	**.15-.22**	**.22-.3**
$\alpha_s(60)$	*.198*	*.095*	*.125*	*.148*	*.142*	*.134*	*.125*	*.114*	*.106*	*.119*	*.129*
$\alpha_s(91)$	*.191*	*.098*	*.121*	*.144*	*.135*	*.142*	*.129*	*.105*	*.091*	*.112*	*.127*
$\alpha_s(133)$	*.144*	*.116*	*.135*	*.132*	*.113*	*.148*	*.124*	*.095*	*.136*	*.131*	*.138*
$\alpha_s(177)$	*.090*	*.138*	*.116*	*.127*	*.080*	*.150*	*.117*	*.100*	*.083*	*.119*	*.134*
$\alpha_s(197)$	*.094*	*.092*	*.113*	*.128*	*.100*	*.082*	*.121*	*.109*	*.094*	*.148*	*.140*

6. Conclusion

In this paper we considered three jet rate and thrust at NNLO. First by considering LO and NLO calculations of strong coupling constant we showed that these calculations aren't exact and we need to use of NNLO. Then we tested NNLO with different methods. After that three jet observables and thrust at NNLO were considered. The predictions of the NNLO were found to be in general agreement with the measured distributions. In general, NNLO provides the best description of the data and LO the least good .From the three jet and thrust distributions, we extracted the strong coupling constant, α_s, and test it's evaluation with different centre of mass energies. The results were consistent with the running of α_s, expected from QCD predictions and ξ spectrum.

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