Semi- analytical investigation of a transverse magnetic field on Viscous Flow over a Stretching Sheet

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Abstract: In this article, Homotopy Analysis Method (HAM) is applied to solve the nonlinear equation of MHD Viscous Flow over a Stretching Sheet. The obtained results were compared with Numerical solution to verify the preciseness of the proposed method. Finally, the influence of the strength of nonlinearity (β) and the Hartman electric number (Ha) on velocity profiles are debated and depicted graphically.

Keywords: MHD Viscous Flow; Stretching Sheet., Homotopy analysis method

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1. Introduction

The term of Magnetohydrodynamic (MHD) wasfirstly introduced by Alfvén[1]. The theory of Magnetohydrodynamics is inducing current in a moving conductive fluid in the presence of magneticfield; such induced current results force on ions of the conductive fluid. The theoretical study of (MHD) channel has been a subject of great interest due to its extensive applications indesigning cooling systemswith liquidmetals, MHD gener-ators, accelerators, pumps andflow meters[2]

Nonlinear phenomena, which appear in many scientific fields, such as plasma physics, solid state physics, chemical kinetics and fluid mechanics, can be modeled by solving nonlinear boundary layer differential equations. The boundary layer flow of an in-compressible viscous fluid over a continuously stretch-ing sheet is often encountered in many engineering and industrial processes. Investigations of boundary layer flow of viscous fluids over a stretching sheet are important in many manufac turing processes such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, glass-fiber, metal extrusion and metal spinning[3,4].

In most cases, these problems do not admit analytical solution, so these equations should be solved using special techniques. In recent years, much attention has been devoted to the newly developed methods to construct an analytic solution of equation; such as method include the Perturbation techniques. Perturbation techniques are too strongly dependent upon the so-called ‘‘small parameters’’ [5]. Other many different methods have introduced to solve nonlinear equation such as homotopy perturbation [6–8] and homotopy analysis methods[9-15], variational iteration method [16-19] and Exp-Function method [20].

In this letter, analytical solution of nonlinear equation arising of viscous incompressible fluid in stretching sheet has been studied by HAM. Obtaining the analytical solution of the models and comparing with numerical result reveal the capability, effectiveness and convenience of HAM. This method gives successive approximations of high accuracy solution.

The numerical solution is performed using the algebra package Maple 16.0, to solve the present case. The software uses a second-order difference scheme combined with an order bootstrap technique with mesh-refinement strategies: the difference scheme is based on either the trapezoid or midpoint rules; the order improvement/accuracy enhancement is either Richardson extrapolation or a method of deferred corrections [21]. This method gives successive approximations of high accuracy solution.

2. Discription of the problem

Let us consider the steady two-dimensional MHD flow of an incompressible viscous fluid over a nonlinear porous shrinking sheet at $y=0$. The fluid is electrically conducted under the influence of an applied magnetic field $B(x)$ normal to the stretching sheet.The induced magnetic field is neglected. The continuity and momentum Equations describing the flow can be written as[22, 23]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}=\nu\,\frac{\partial^2 u}{\partial y^2}-\sigma\frac{B_0^2(x)}{\rho}u \tag{2}$$

Here u and v are the velocity components in the xand y directions, respectively. Also υ, ρ and σ are the kinematic viscosity, density and electrical conductivity of the fluid. In Equation (2), the external electric field and the polarization effects are negligible, and:

$$B(x)=B_0\;x^{\frac{(n-1)}{2}} \tag{3}$$

The appropriate velocity boundary conditions become:

$$\begin{aligned}&u(x,0)=c\,x^n, \qquad v(x,0)=0\\&u(x,u)\to 0,\quad y\to\infty\end{aligned} \tag{4}$$

In order to solve the problem, momentum and energy equations are firstly nondimensionalized by using the following similarity variables:

$$\eta=\sqrt{\frac{c(n+1)}{2\upsilon}}x^{\frac{n-1}{2}}y\;, \qquad u=c\;x^n\;\;f'(\eta) \tag{5}$$

$$v=\sqrt{\frac{c(n+1)}{2\upsilon}}x^{\frac{n-1}{2}}\left[\,f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right] \tag{6}$$

Equations (1)-(4) are transformed to

$$f'''(\eta)+f(\eta)f''(\eta)-\beta f'(\eta)^2-M\;f'(\eta)=0 \tag{7}$$

With following boundary conditions:

$$f(0)=0, \qquad f'(0)=1, \qquad f'(\infty)=0 \qquad (8)$$

Where

$$\beta=\frac{2n}{n+1}, \qquad M=\frac{2\sigma}{\rho c(1+n)} \qquad (9)$$

In the next sections we shall solve the system of Equations. (12) by using the HAM.

3. Analytical applied methods

For HAM solutions, we choose the initial guesses and auxiliary linear operators in the following form:

$$f_0(\eta)=1-e^{-\eta}, \qquad (10)$$

$$L(f)=f'''+f'', \qquad (11)$$

$$L(C_1+C_2\eta+C_3e^{-\eta})=0, \qquad (12)$$

And $c_i(i=1-3)$ are constants. Let $P\in[0,1]$ denotes the embedding parameter and $\hbar$ indicates non–zero auxiliary parameters. We then construct the following Equations:

Zeroth –order deformation Equations

$$(1-P)L_1\left[f(\eta;p)-f_0(\eta)\right]=p\hbar_1 H(\eta)N_1\left[f(\eta;p)\right] \qquad (13)$$

$$f(0;p)=0; \qquad f'(0;p)=1; \qquad f'(\infty;p)=0 \qquad (14)$$

$$N\left[f(\eta;p)\right]=2\frac{d^3f(\eta;p)}{d\eta^3}+f(\eta;p)\frac{d^2f(\eta;p)}{d\eta^2}-\beta\left(\frac{df(\eta;p)}{d\eta}\right)^2 -M\frac{df(\eta;p)}{d\eta}=0, \qquad (15)$$

For $p=0$ and $p=1$ we have

$$f(\eta;0)=f_0(\eta) \qquad f(\eta;1)=f(\eta) \qquad (16)$$

When p increases from 0 to 1 then $f(\eta;p)$ vary from $f_0(\eta)$ to $f(\eta)$. By Taylor's theorem and using Equations (16), $f(\eta;p)$ can be expanded in a power series of p as follows:

$$f(\eta;p)=f_0(\eta)+\sum_{m-1}^{\infty}f_m(\eta)p^m, \quad f_m(\eta)=\frac{1}{m!}\frac{\partial^m(f(\eta;p))}{\partial p^m} \qquad (17)$$

In which $\hbar_1$ and $\hbar_2$ is chosen in such a way that these two series are convergent at $p=1$, therefore we have through Equations (17) that

$$f(\eta)=f_0(\eta)+\sum_{m-1}^{\infty}f_m(\eta), \qquad (18)$$

mth –order deformation Equations

$$L_1\left[f_m(\eta)-\chi_m f_{m-1}(\eta)\right]=\hbar_1 H(\eta)R_m^f(\eta) \qquad (19)$$

$$f_m(0)=0; \qquad f_m'(0)=0; \qquad f_m'(\infty)=0, \qquad (20)$$

$$R_m(\eta)=2f_{m-1}'''(\eta)+\sum_{n=0}^{m-1}\left[f_{m-1-n}(\eta)f_n''(\eta)-\beta f_{m-1-n}'(\eta)f_n'(\eta)\right] -Mf_{m-1}'(\eta) \qquad (21)$$

Now we determine the convergency of the result, the differential Equation, and the auxiliary function according to the solution expression. So let us assume:

$$H(\eta)=e^{-\eta} \qquad (22)$$

We have found the answer by maple analytic solution device. Two first deformations of the coupled solutions are presented below.

$$f_1(\eta) = {}^{-\eta}\left(-\frac{1}{6}\hbar\beta - \frac{1}{2}\hbar M^2 + \frac{1}{6}\hbar\right) + \frac{1}{18}\hbar^{-3\eta}\beta + \frac{1}{4}\hbar^{-2\eta}M^2 - \frac{1}{18}\hbar^{-3\eta}$$
$$+\frac{1}{9}\hbar\beta + \frac{1}{4}\hbar M^2 - \frac{1}{9}\hbar \qquad (23)$$

The solutions $f_{10}(\eta)$ were too long to be mentioned here, therefore, they are shown graphically.

4. Convergence of the HAM solution

As pointed out by Liao, the convergence region and rate of solution series can be adjusted and controlled by means of the auxiliary parameter ħ. In general, by means of the so-called ħ-curve, it is straightforward to choose an appropriate range for ħ which ensures the convergence of the solution series. To influence of ħ on the convergence of solution, we plot the so-called ħ-curve of $f''(0)$ by 11th-order approximation, as shown in Figure 1.

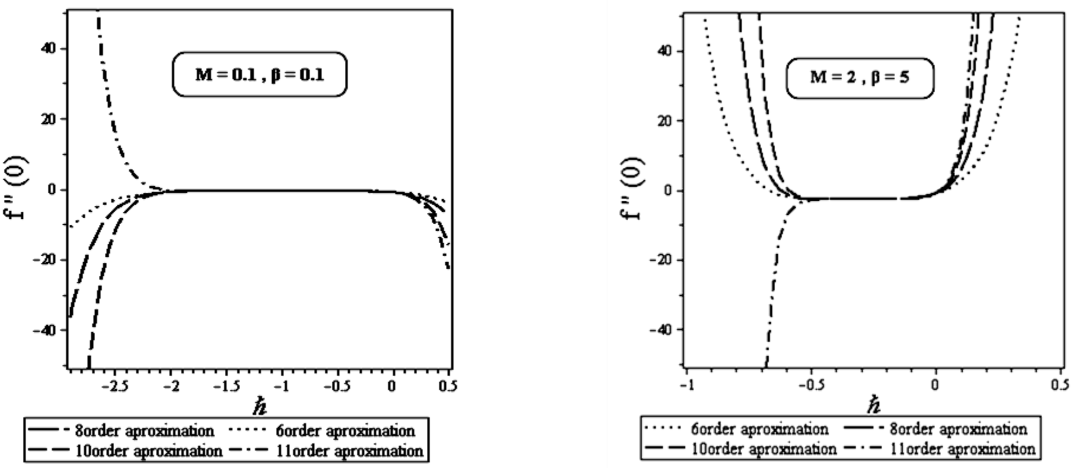

Figure 1. *The* $\hbar$ *- validity of* $f''(0)$ *for different value of* β *and* M

The solutions converge for ħ values which are corresponding to the horizontal line segment in ħ curve.

5. Results and discussion

In this study, the problem of magneto-hydrodynamic flow over a nonlinear stretching sheet was analyzed using HAM. The effects of active parameters such as the nondimensional parameter and Hartmann number on flow are investigated. The present code is validated by comparing the obtained results with the numerical method (four-order Runge-Kutta) for different values of active parameters. Tables 1 and 2 show the comparison of HAM results with numerical RK-4 for different values of parameters. It is noteworthy to mention here that the low error of HAM is remarkable, while the effectiveness of the proposed method (HAM) can be concluded from Figures 3 and 4, where we have presented the effect of parameters. All these comparisons illustrate that HAM offers a highly accurate solution for the present problem. Moreover, Figs. 3 and 4 are prepared in order to see the effects of the physical parameters on the velocity components. The effect of parameters on dimensionless velocity profile is observed in Figure 2, in which the velocity profile decreases with the increase in magnetic parameter by keeping fixed the other parameter. In addition, it can be pointed out that, as the parameters of β increase, the velocity descend over the stretching sheet as shown in Figure 3.

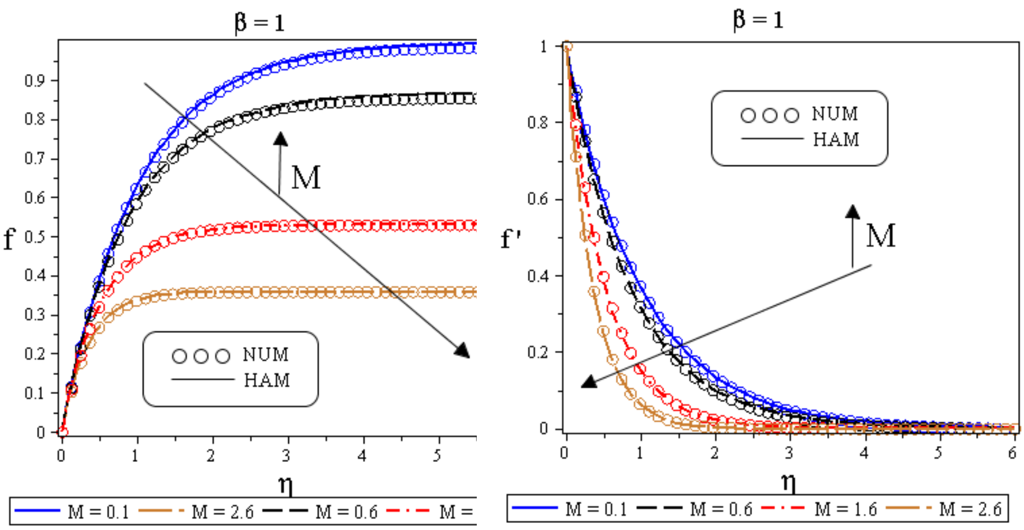

Figure 2. *Effect of Hartman electric number M and comparison of HAM and numerical method (NUM) in different M number when $\beta = 1$ and $\hbar = -0.5$*

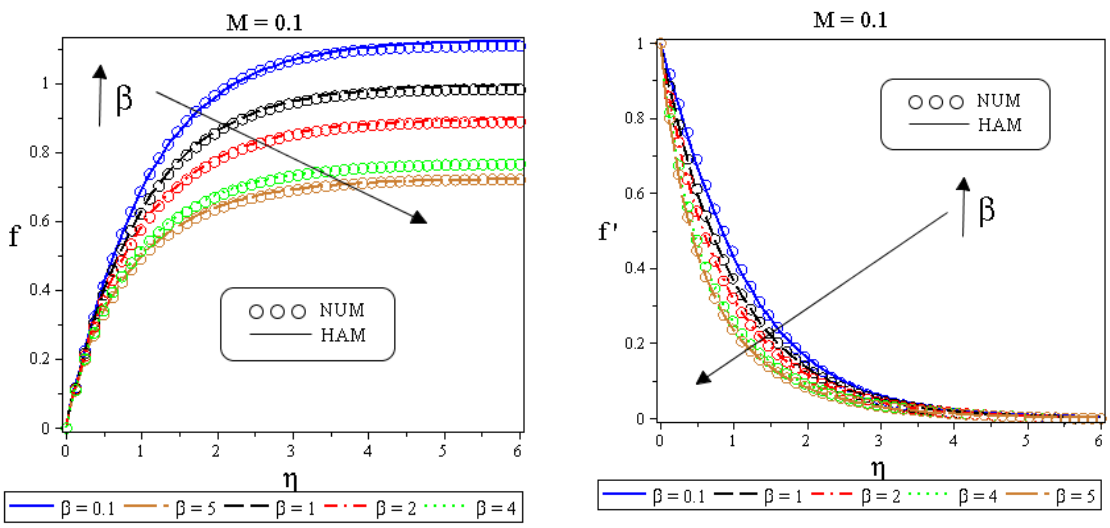

Figure 3. *Effect of nonlinearity number β and comparison of HAM and numerical method (NUM) in different β number when $M = 0.1$ and $\hbar = -0.5$*

Table 1

The results of HAM solution and Numerical methods for $f(\eta)$ and $f'(\eta)$ for $M = 1.5, \beta = 0.9$

η	$f(\eta)$			$f'(\eta)$		
	HAM	NUM	Error	HAM	NUM	Error
).00	0.000000000	0.000000000	0.000000000	1.000000000	1.000000000	).000000000
).05	0.0478335389	0.0478337436	2.04710E-07	0.914598704	0.914603907	5.20300E-06
).10	0.0915802774	0.0915809362	6.58820E-07	0.836423579	0.836434332	1.07538E-05
).	0.131586289	0.131587629	1.34070E-06	0.764873574	0.764890327	1.67532E-

15						05
0.20	0.168168817	0.168171131	2.31340E-06	0.699396005	0.699419117	2.31118E-05
0.25	0.201618608	0.201622231	3.62340E-06	0.639482939	0.639512664	2.97246E-05
0.30	0.232202055	0.232207340	5.28450E-06	0.584667799	0.584704286	3.64866E-05
0.35	0.260163194	0.260170486	7.29200E-06	0.534522199	0.534565458	4.32595E-05
0.40	0.285725530	0.285735163	9.63260E-06	0.488652978	0.488702829	4.98515E-05
0.45	0.309093737	0.309106024	1.22869E-05	0.446699450	0.446755459	5.60090E-05
0.50	0.330455208	0.330470436	1.52280E-05	0.408330829	0.408392259	6.14295E-05
0.55	0.349981500	0.349999913	1.84135E-05	0.373243850	0.373309629	6.57795E-05
0.60	0.367829642	0.367851426	2.17842E-05	0.341160544	0.341229258	6.87143E-05
0.65	0.384143358	0.384168619	2.52607E-05	0.311826196	0.311896091	6.98947E-05
0.70	0.399054177	0.399082923	2.87466E-05	0.285007441	0.285076439	6.89975E-05
0.75	0.412682456	0.412714583	3.21275E-05	0.260490509	0.260556240	6.57309E-05
0.80	0.425138319	0.425173598	3.52784E-05	0.238079608	0.238139447	5.98396E-05
0.85	0.436522520	0.436560584	3.80632E-05	0.217595430	0.217646546	5.11163E-05
0.90	0.446927230	0.446967568	4.03377E-05	0.198873782	0.198913192	3.94103E-05
0.95	0.456436761	0.456478713	4.19518E-05	0.181764319	0.181788952	2.46326E-05
1.00	0.465128231	0.465170982	4.27505E-05	0.166129391	0.166136152	6.76120E-06

Table 2
The results of HAM solution and Numerical methods for $f(\eta)$ and $f'(\eta)$ for $M=2, \beta=0.5$

η	$f(\eta)$			$f'(\eta)$		
	HAM	NUM	Error	HAM	NUM	Error
0.00	0.0000000000	0.0000000000	0.00000000	1.00000000	1.000000000	0.00000000
0.05	0.0473903596	0.0473904195	5.9910E-08	0.89740145	0.897400877	5.7760E-07
0.10	0.0899104028	0.089910448	4.5150E-08	0.80502107	0.805019969	1.1040E-06
0.15	0.128046769	0.128046721	4.8300E-08	0.72190139	0.721899762	1.6323E-06
0.20	0.162240188	0.162240022	1.6590E-07	0.64716292	0.647160550	2.3705E-06
0.25	0.192889283	0.192888979	3.0490E-07	0.58000013	0.579996727	3.4013E-06

0.30	0.220354162	0.220353668	4.9310E-07	0.51967716	0.519672426	4.7342E-06
0.35	0.244959785	0.244959025	7.5980E-07	0.46552335	0.465516971	6.3768E-06
0.40	0.266999118	0.266997994	1.1244E-06	0.41692869	0.416920335	8.3597E-06
0.45	0.286736048	0.286734449	1.5997E-06	0.37333940	0.373328685	1.0716E-05
0.50	0.304408088	0.304405888	2.2001E-06	0.33425350	0.334240045	1.3458E-05
0.55	0.320228859	0.320225915	2.9442E-06	0.29921668	0.299200116	1.6565E-05
0.60	0.334390374	0.334386522	3.8520E-06	0.26781827	0.267798283	1.9986E-05
0.65	0.347065126	0.347060186	4.9397E-06	0.23968749	0.239663837	2.3650E-05
0.70	0.358407991	0.358401773	6.2178E-06	0.21448992	0.214462442	2.7475E-05
0.75	0.368557966	0.368550275	7.6904E-06	0.19192421	0.191892841	3.1365E-05
0.80	0.377639739	0.377630385	9.3546E-06	0.17171903	0.171683821	3.5213E-05
0.85	0.385765129	0.385753923	1.1206E-05	0.15363029	0.153591398	3.8892E-05
0.90	0.393034372	0.393021136	1.3235E-05	0.13743850	0.137396230	4.2269E-05
0.95	0.399537297	0.399521872	1.5425E-05	0.12294645	0.122901235	4.5218E-05
1.00	0.405354384	0.405336634	1.7750E-05	0.10997705	0.109929424	4.7623E-05

6. Conclusions

In this article, we have successfully applied the homotopy analysis method for MHD viscous flow over a stretching sheet. Both numerical and approximate analytical results are obtained for the problem. The results obtained via HAM are presented in tabular and graphical forms. According to Tables 1, 2 and Figs 3, 4 clearly show that the results by HAM is in excellent agreement with the exact solutions. Also, it is expected here that this powerful mathematical tool can solve a large class of nonlinear differential systems, especially nonlinear equations used in engineering and physics.

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