Production Cost Management under Loop and re-work Conditions Using GERT Network

Saeed Razeghi[1], Soleiman Iranzadeh[2], Kamaleddin Rahmani Youshanloee[3], M. Bagherpour[4], Hossein Bevrani[5]

[1] Department of Industrial Management, East Azarbaijan Science and Research Branch, Islamic Azad University, Tabriz, Iran
[2] *Department of Management, Tabriz Branch, Islamic Azad University, Tabriz, Iran
[3] Department of Management, Tabriz Branch, Islamic Azad University, Tabriz, Iran
[4] Department of Industrial Engineering, Shiraz branch, Islamic Azad University, Shiraz, Iran
[5] Department of Statistics, Faculty of Mathematical Sciences, university of Tabriz, Tabriz, Iran
*Corresponding author

Abstract: Cost management using Earned Value Management (EVM) has been extensively applied by many researchers. However, the EVM requires a reliable schedule which in standard EVM loop and re-works can not taken into EVM based calculations. However, in real case projects, re-works is often un-avoidable. Therefore a need for developing a cost management model under re-work is highly desirable which is considered in this paper. Also, in this research a well organized mechanism for managing production costs has developed where loop and re-works are un-limitedly allowed. The approach is implemented through a case and the results show a robust production cost management system which is compared with the deterministic models.

Keywords: production costs, cost management, GERT network, Earned Value management.

Introducion

Production processes consists of some operations to be analyzed for planning and forecasting. Production system, accompanied by the transposition of each production operation and each activity has its own time and cost for producing each unit of the final product. In fact, with the arrival of a production machine, semi-finished materials pass through them with a certain probability and involve in production route. Meanwhile, some percentage of parts is formed as wastes, which is either considered as wastes and/or remain in the system for duplications. During production, time and cost of an operation lead to some estimations for the final productions that will have considerable impacts on scheduling and planning products and further management decisions. Variable production conditions cause time and cost not to be considered as accurate and deterministic factors. unfortunately, the data obtained from productions show that excess of costs were brought about as a quite normal task rather than as an exception [1]. In case production orders in a system are considered as a project-oriented perspective, scheduling and appropriate use of resources during order preparation will be considered as the planning principles. On implementing a project within a specified interval, activities are considered under considerable uncertainty conditions, as they lead to numerous scheduling problems and moving toward them [2]. Such uncertainty is caused with respect to the conditions that may include increasing and/or decreasing runtime of each activity compared to the preliminary estimation and/or having no access to the required resources and/or obtaining raw materials later than the time specified [2]. In executive processes of each system, it is important to pay attention to cost factor along with scheduling factor and various studies discussed the relationship between these two factors. In real projects, tradeoff among project completion time, its administrative cost, and environmental uncertainty are among the remarkable aspects for decision-makers [3]. Presence of environmental uncertainty in the two indices discussed in productions and projects complicated the issues related to these factors greatly. Relations of transposition and duplication in the production and project systems and efficiency factor and/or probability in any operation considered the use of some networks in uncertain forms. Network techniques are important tools for scheduling projects, as they are able to show problems or insignificant impacts in execution of projects while project networks are complex and time of activities are uncertain [4]. While robust tools for planning and controlling projects are

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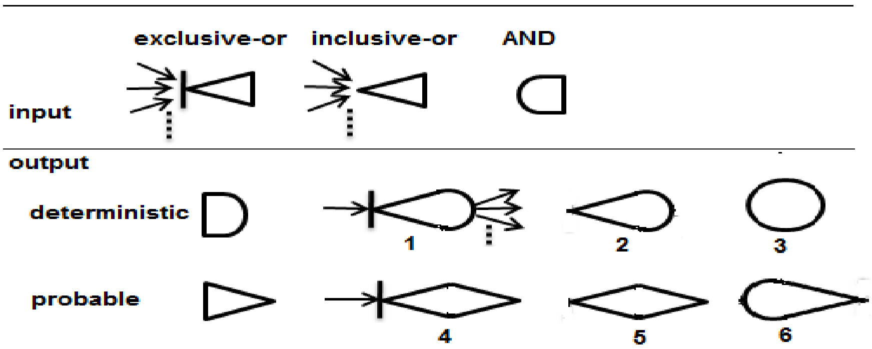
available, they have some advantages including creation a stable framework for planning, scheduling, supervision and project control. They indicate dependencies of project activities [5]. Regarding appropriateness of the GERT network model in such probability issues reduced the complexity to some extent; it will provide appropriate results for decision-makers and managers. GERT is an important tool for analyzing progress of major projects, managerial decisions, production scheduling and reliability. In current research methods, it is able to solve and analyze the issues related to duplication phenomenon effectively [6]. This technique is a method made up of the flow graph theory, PERT, and the moment-generating function for solving probability issues. The GERT technique is in fact a network-based approach, which is very similar to PERT method. With respect to the properties of such networks with nodes having different properties, we may executive indefinite calculations even within the form of R&D projects, which deal with a high level of uncertainty [7]. Generally, there are two approaches for solving GERT problems, which include analytic and simulation approaches. Here, the existing exclusive-or nodes and inclusive-or nodes in a network is studied with respect to an analytic approach and a simulation approach, respectively. In all systems, the presence of planning and control to achieve the relevant objectives is very crucial success factor. In manufacturing environment, planning topics attracted attentions and different studies were conducted in this concern during several years. Apparently, focus on planning concepts of production and manufacturing exceeded designing an organized controlling system, as it enabled producers to manage manufacturing and production processes [8]. It should be noted that paying attention to the composition of planning discussions, that is predictions before real execution and controlling mechanism during execution, would lead to useful results. The earned value management (EVM) technique is among the effective controlling tools in project-based systems. The known EVMS1 is a managerial system that combines cost, scheduling and technical performance, calculates cost and time deviations and performance indices, and predicts time and cost of a project [9]. Meanwhile, diagram S in EVMS acts as the main factor in the relationship between scheduling and project costs [1]. As there is a close relationship between cost evaluation and budgeting of each project, paying attention to scheduling, cost (as compared with the preliminary budgeting) and performance in project-based systems is of crucial importance. It should be noted that in projects, due to specific reasons, achieving the above three factors is encountered by some problems. The problems include lack of stating demands appropriately, weak scheduling, inappropriate technical skills and coherence, lack of teamwork, weak relationships and arrangement, inefficient supervision over progress and inappropriate support [10]. Regarding coherence, stability and memory centrality, earned value calculations is clear, it is more flexible, and it plays an appropriate guiding role in modern measurements for scope of insight on standard and advanced quantities [11]. As time and cost are important in production and project systems, it should be stated that one of the major challenges in the managerial topic of projects is to maintain balance in the three concepts of scope, scheduling, and cost [12]. Because it is of crucial importance for all managers to pay attention to managing the projects with low cost and short time and having reliable prediction tools [13]. The ratios obtained from EVM calculations are very important for future planning of a system, as planners will make more real predictions within 2 main indices of time and cost and a combination of them. Meanwhile, planning charts, such as Gant – that briefly shows an overall view of execution operation in a form – can be considered. Lack of access of executers to early predictions of time and cost occurs due to different reasons that may result in negative and positive impacts. Such impacts – appropriate or inappropriate to objectives and performance - are expressed as risks. In projects, risks are usually bought about with respect to possibility of the events, which have inconsistent impacts on objectives and performance of a project. Positive risks are also expressed as opportunities with positive impacts on achieving objectives of a project [14]. Paying attention to the real causes of risk occurrence regarding objectives is a useful task for better guarantee of execution of work from the perspective of mangers. Generally, four groups of risk factors are considered in projects for concentration of project managers, which include risk of technical performance, risk of scheduling, risk of cost, and risk of execution [10]. Most findings indicate that clearing and risk-taking ability will have direct impacts on success of a project portfolio [15].

2. Research methodology

[1] Earned Value Management System

After determination of operations and relationships, the executive network would be plotted. This network includes all required operation and the possibilities for each of them. It is required in GERT network to consider the main parameters for each of the operations, the existence of certainty and uncertainty in them. The use of moment-generating functions for analytic calculations in uncertain conditions is required as well. While plotting such networks, the calculation method is determined considering the knot input and outputs and definition of each one. Logical nodes in such networks consist of two input and output sections.

Table1 : GERT node symbol



Flow graph is a graphical representation of the relationships between the variables in they are all shown simultaneously. Therefore, nodes represent the variables and arcs represent a transfer function (in form a numeral value, a parameter, an integral or a derivative). Consider that in a network, each node can be spotted as a linear combination of all arcs that enter it.

$$\begin{cases} x_2 = ax_1 \\ x_3 = bx_1 + d \\ x_4 = cx_3 + e \end{cases} \quad (1)$$

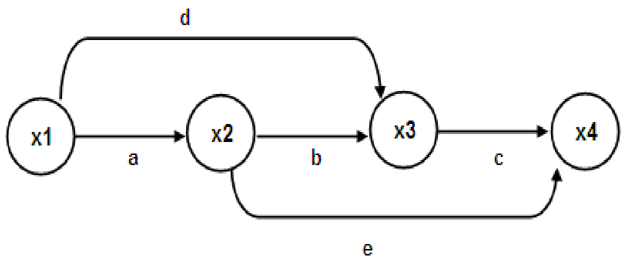


Fig.1.network model

Considering the calculations in figure 1, the value of displacement parameter from X1 to X4 nodes is specified based on the linear combination of nodes and if the regulations of uncertain GERT (Mason) networks are applied, the foresaid equation will be resulted as well. The network topology is determined through applying Mason's rules in network calculations.

$$w_{0-n} = \frac{\sum \text{value of path}_i \times \text{topology}\,(\text{untangent}_i)}{\text{total topology}} \quad , \quad \text{total topology} = 1 + \sum_m \sum_i (-1)^m L_i\,(m) \qquad (2)$$

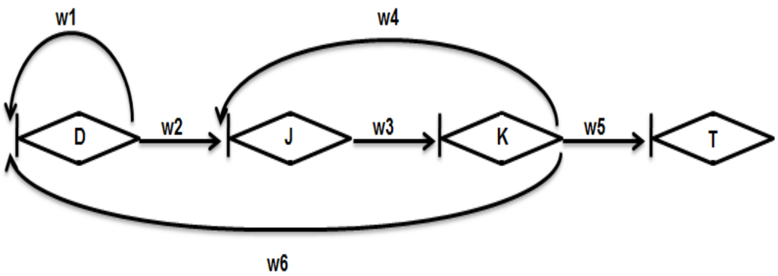


Fig.2.GERT network parametric model

The fact that the weight value (Wij) for each arc is equal to the product of its probability and the moment-generating function of the considered variable (variables), therefore:

$$\begin{cases} W_{ij} = P_{ij}.\left(\prod_{n}^{i=1} M_{ij}(s)\right),\ W_{D-T} = \dfrac{(w_2.w_3.w_5)}{1-(w_1+w_3.w_4+w_2+w_3+w_6)+(w_1.w_3.w_4)} \\ P_{D-T} = W_{D-T}(\forall_s = 0) \\ M_{D-T}(s) = \dfrac{W_{D-T}(s)}{P_{D-T}} = M_{D-T}(s) = \dfrac{W_{D-T}(s)}{P_{D-T}} \end{cases} \quad (3)$$

$$\frac{\partial M_{D-T}(s)}{\partial s}_{(s.to:\forall s=0)} \rightarrow \mu'_1 = \mu \quad , \quad \frac{\partial^2 M_{D-T}(s)}{\partial s^2}_{(s.to:\forall s=0)} \rightarrow \mu'_2 \quad , \quad \sigma^2 = \mu'_2 - \mu^2 \quad (4)$$

If the linear relations between the node and the arc were applied instead of GERT analysis, the resulted weight value would be the same as it is indicated in Mason's method. It is required to develop multiplicative relations in calculation of linear and Mason relations and in uncertain conditions, moment-generating functions are applied in order to convert addible parameters into multipliable ones.

3. EVM (Earned Value Management) methodology:

Earned value management is one of the useful techniques in cost control system. This is one of the general and well-known management methods, which combines the cost scheduling and technical performance and makes it possible to calculate time and cost deviations, determine performance indicators and predict the needed time and cost for the project. Progress of activities, and the time and costs spent for complementation of the project are considered in this technique in which the risk of project is evaluated and controlled through measuring its monetary progress [16]. It should be mentioned that cost controlling systems have been greatly improved upon recent years there has been many studies even in uncertain and unconfident conditions [8]. Three main indicators are considered in calculation of the earned value. Reciprocity and comparison of these factors can be applied in determination of variations, calculation of performance indicators and prediction of cost and time.

1. Budgeted Cost for Work Scheduled (BCWS) or Planned Value (PV): Aggregate values of the predicted according to the distributed budget on schedule.

$$BCWS = PV = \sum_{i=1}^{n} Budget_i * \%\ planned\ progress_i \quad (5)$$

2. Budgeted Cost of Work Performed (BCWP or EV): Aggregate values of the predicted budget for the actual plan up to a particular time based on the actual progress of the performed tasks.

$$BCWP = EV = \sum_{i=1}^{n} Budget_i * \%\ actual\ progress_i \quad (6)$$

3. Actual Cost of Work Performed (ACWP or AC): Total costs spent directly or indirectly up to a specific time.

$$ACWP = AC = \sum_{i=1}^{n} Cost_i \quad (7)$$

Considering three foresaid factors, Time and cost variations and performance indicators can be calculated in order to be used for predictions.

$$CV_{tn} = BCWP_{tn} - ACWP_{tn}\ ,\ SV_{tn} = BCWP_{tn} - BCWS_{tn}\ ,\ SPI_{tn} = \frac{BCWP_{tn}}{BCWS_{tn}}\ ,\ CPI_{tn} = \frac{BCWP_{tn}}{ACWP_{tn}} \quad (8)$$

{sv=schedule variation,cv=cost variation}

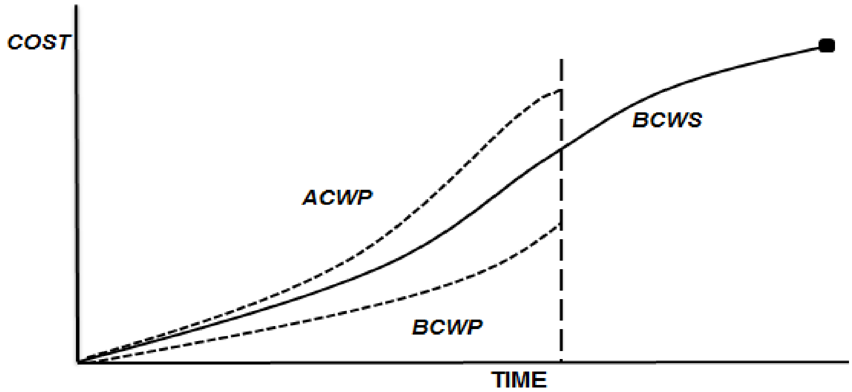


Fig.3.comulative planned value, Earned value and actual cost curve

Prediction of the total cost of project (Estimate at completion): Estimation of the actual cost of project considering its present progress.

$$EAC_{tn} = Actual\ cost_{tn} + ETC_{tn} \quad (9)$$

$$ETC_{tn} = \frac{BAC - BCWP_{tb}}{PF_{tn}} \quad (10)$$

{BAC (budget at completion or the project total cost) , PF (performance factor)}

tn= time now, PF= CPI (or) CPI*SPI

4. The comparative calculations of production lines including the loop (rework):

It is important to consider linear relationships of inputs and outputs in production networks using rework operations are usually performed. For instance, if you intend to calculate system efficiency, you can easily analyze it through linear methods, even including reworks. If the rework loops have no limitation in repetition (and they can be repotted for ever) progression should be considered in calculations as well. The following production line is considered in which each multiplicative factor (α) can be analyzed through Mason’s rule considering the number of loops and its repetitions along the entire line. The following production system includes one loop and if a set of materials enters this system, the number of incidents in the loop would be different according to the system definition. If each operation is defined based on the efficiency of an executive system, considering the wasted amount and a percentage that returns the system for iteration, no material remains for repeating the loop after several uploads. i.e. the waste materials are being recycled as far as possible. It should be mentioned that each multiplicative value can refer to probability, efficiency, or the moment-generating function (time, cost, etc) of any operation. About the loops in uncertain networks, tangency and non-tangency should be considered as well which complicates the Mason calculations. Loops indicating the recurrence operations in the system are happening with a specified probability and their occurrence makes the system path longer. Loops are repeatable operations in which the probability specifies the number of repetitions. If any of loop happens more than once, the possibility of its occurrence would decrease for a piece in each repetition compared to previous occurrence. Considering the number of loop repetitions, networks which contain the loops can be converted to loop-less networks. The number of these repetitions is indicated in form of system hypotheses considering the probability of each operation and its moment-generating functions. After determining the number of repetitions for each loop, new loop-less networks would be developed through opening the loops considering the system definition where comparative calculations would be performed in order to prove the equality of the new network and the previous one.

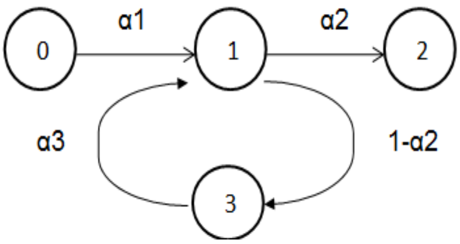


Fig.4. network with loop

Rule 1: If the loop above happens for once, the representative form of the loop can be opened and shown this way:

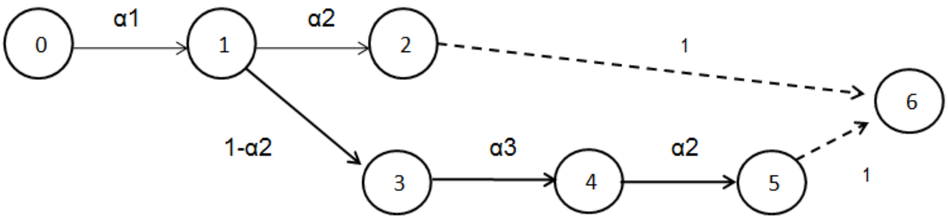


Fig.5. network without loop

Figure 5 shows a production line with no loop which can apply Mason’s rule or linear combination in order to calculate the earned value from the beginning u to the end of the line.

$$w_{0-n} = \frac{\sum \text{value of path}_i \times \text{topology}(\text{untangent}_i)}{\text{total topology}} \quad (11)$$

$$w_{0-6} = (\alpha 1 \times \alpha 2 \times 1) + (\alpha 1)(1 - \alpha 2)(\alpha 3)(\alpha 2 \times 1) \quad (12)$$

Rule 2: if the loop in figure 4 happens twice, the representative form of the loop can be opened and shown this way:

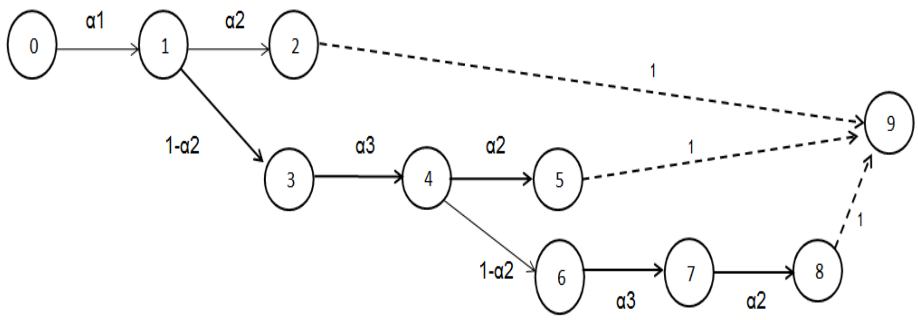


Fig.6. network without loop

$$W_{0-9} = (\alpha 1 \times \alpha 2 \times 1) + (\alpha 1)(1 - \alpha 2)(\alpha 3)(\alpha 2 \times 1) + (\alpha 1)(1 - \alpha 2)(\alpha 3)(1 - \alpha 2)(\alpha 3)(\alpha 2 \times 1) \quad (13)$$

Rule 3: if the loop goes on to infinity, the representative form of the loop can be opened and shown for infinite times. In fact, the term infinite refers to the number of loop implementation up to the point that is allowed by the probability or efficiency of the system. Actually, each time that the loop is repeated for one piece with specific probability, the probability of its occurrence decreases and tends to zero.

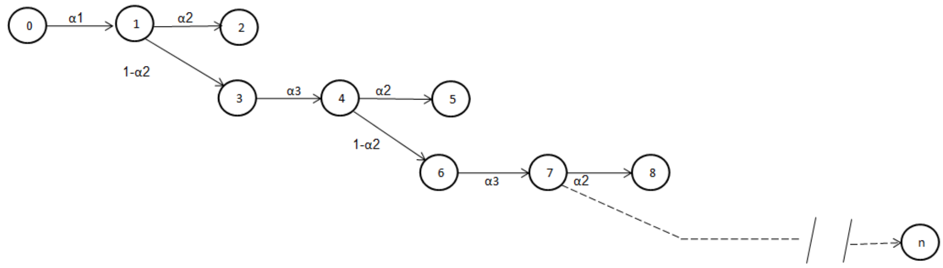


Fig.7. network without loop

$$R = (\alpha 1 \times \alpha 2 \times 1) + (\alpha 1)(1 - \alpha 2)(\alpha 3)(\alpha 2 \times 1) + \cdots . + {\alpha_n}^k (1 - \alpha_{rn})^k (1 - \alpha_n) + \cdots \quad (14)$$

The formula above develops a geometric progression with a relative magnitude smaller than 1, therefore:

$$\lim_{k \to \infty} R = \frac{\alpha 1 \times \alpha 2}{1 - [\alpha_3 (1 - \alpha_2)]} \quad (15)$$

Mason's formula for determination of network value, considering the existence of loops, is calculated below. Since the resulted equation is the same as equation (15), both networks are similar in value.

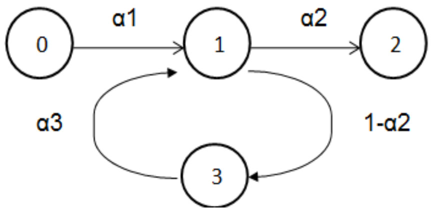


Fig.8. network with loop

$$(\text{mason})\, w_{0-2} = \frac{\alpha 1 \times \alpha 2}{1-[\alpha_3(1-\alpha_2)]} \quad (16)$$

As it can be observed, in forms which contain loops, Mason's rule can be applied without opening the loops when they are repeated to infinity. Considering the fact that the loops can be opened up to n times, they can convergence to Mason's calculations. i.e. if you apply Mason's rule to resolve a network through opening its loops for infinite times (considering the logical value of the occurrence of each loop and its reduction in the next round), the answers would be similar to the condition in which loops are not opened.

5. Earned value management in repetitive networks:

Considering the significance of the Earned value calculations in order to investigation of operation progress status and the existence of probability factor, a comparative approach is presented based on GERT theory. The existence of iteration and uncertainty in this process is among the complicated situations. Considering their probabilities, the loops would be opened as far as possible with an insignificant error, and the analysis would be performed through plotting Gantt production network. Through investigating the progress and combination of time and cost distribution functions in considered dates in the program, true performance and development of new generating functions, the possibilities would be determined in an interval and indicators would be calculated. Considering the presence of numerical ranges in calculations of earned value, there would be optimistic and pessimistic values in calculation of indicators. The applied steps in present research are summarized below.

Step1: Determination of production activities, reworks and performance probability for each activity in order to be plotted inform of a network.

Step 2: Determination of time and cost distribution functions for all activities including reworks

Step 3: Determination of time and cost moment-generating function for activities and combination of each one with their probabilities in order to determine the value of each activity

Step4: Calculation of Mason's rule in the network considering time and cost variables and determination of the average value, variance and probability of the production o final product

$$W_{T(Total)} = \frac{\sum \text{value of path}_i\,(c\&t) \times \text{topology}(\text{untangent}_i)}{\text{total topology}} \quad , \quad \text{total topology} = 1 + \sum_m \sum_i (-1)^m L_i(m) \quad (17)$$

$$\text{Total prob}: P_T = W_{T|\forall(s \text{ and } m=0)} \quad (18)$$

$$M_{T(Total)}((s) \text{and} (m)) = \frac{W_T}{P_T} \quad (19)$$

$$\frac{\partial M_T(s \text{ and } m)}{\partial(s) \text{ and } \partial(m)}\bigg|_{(s \text{ and } m=0)} = E(c) \text{ and } E(t) \;,\; \frac{\partial^2 MT(m)}{\partial m^2}\bigg|_{(m=0)} \rightarrow E(c^2) \quad (20)$$

$$s^2(c) = E(C^2) - E^2(C) \quad (21)$$

Step5: Opening the loops as much as possible, forming a loop-fewer network and calculating Mason's rule in order to determine the total probability, average value and total variance of the discussed variables in the new network, prove the equality of the primary network (which contains loops) with the new one and replace it in calculations and analyses.

Step6: Plotting the Gantt chart for the equivalent network considering the probability and average time of operation in order to be investigated in considered dates

Step7: Determination of investigated dates in production program, formation of new cost moment-generating functions considering the percentage of scheduled time on that date for each activity and its combination with average cost and variance of investigated activities in order to determine PV interval.

$$PV_{tn} = \sum_{i=1}^{n} Budget_{(activity)_i} * \%\,planned\ progress_{(activity)_i} \quad (22)$$

$$x_i \sim N(\mu_i, \partial_i^2)\ , C_i \sim N(\mu_{c_i}, \partial^2{}_{c_i}) \quad (23)$$

$$y = lx \to y_i \sim N(l\mu_i, l^2 \partial_i^2)\ \ , C = lc \to C_i \sim N(l\mu_{c_i}, l^2 \partial^2{}_{c_i})\ ,\ (l = planned\ progress\ of\ activity_i) \quad (24)$$

Step 8: Determination of actual progress percentage for the project on the investigated date and its combination with cost moment-generating functions in order to form the EV interval

$$EV_{tn} = \sum_{i=1}^{n} Budget_{(activity)_i} * \%\,actual\ progress_{(activity)_i} \quad (25)$$

$$C = lc \to C_i \sim N(l\mu_{c_i}, l^2 \partial^2{}_{c_i})\ ,\ (l = actual\ progress\ of\ activity_i) \quad (26)$$

Step9: Determination of (optimistic, probable and pessimistic) scenarios considering the resulted PV and EV intervals at the investigated dates, investigation and calculation of time and cost indicator (SPI, CPI, ETC, EAC) according to the obtained interval and determination of the values of actual spent cost up to the considered date (AC)

Step10: Prediction of total cost considering the performance factor and the selected scenario and determination of the changes in costs on investigation dates

Step 11: Determination and analysis of EV and PV values in certain and uncertain conditions (considering the selected scenario).

It should be mentioned that a combination of time and cost indicators have been applied in determination of performance factor (PF), where other methods can be applied as well. The range analysis and sensitivity study in decision making in order to predict and allocate budget is important for the manager and according to their point of view, the predicted values for costs and budgets will be changeable.

6. Case study:

The following production network is considered with four main and to iteration (loop) operations. The output of each operation is regarded considering the probability of the entrance of work-in-process goods. For all operations including the main ones and reworks, the probable time and cost with average value and variance obtained through simulation study are considered according to the history of operations and their normality. Each operation has a normal distribution function for time and cost variables. In order to perform GERT analysis, their moment-generating functions are shown in the table below.

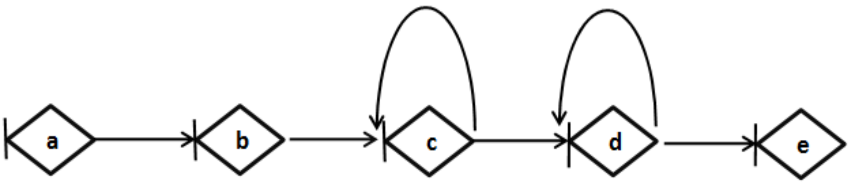


Fig.9.GERT network model

Table2: Activity description, probably, time and cost momentum

activity	Probably	M[s] (time)	M[m] (cost)
a-b	1	$e^{2t+\frac{t^2}{2}}$	e^{7c+c^2}
b-c	1	$e^{5t+\frac{t^2}{2}}$	e^{5c+c^2}
c-d	0.95	$e^{3t+\frac{t^2}{2}}$	e^{15c+3c^2}
c-c	0.05	e^{6t+2t^2}	e^{6c+c^2}
d-e	0.7	e^{3t+t^2}	e^{9c+c^2}
d-d	0.1	e^{4t+t^2}	e^{4c+2c^2}
d-out(spoilage)	0.2	-	-

Considering to the performed control in each operation, the input piece in this system faces some iteration during the production phases, in a way that it returns to the system with a determined probability and a probable time and cost. The probability, average value and variance of time and cost in the whole system(from node a to node e) are calculated applying Mason's analysis.

$$w_{a-e(s)} = \frac{\left(e^{2s+\frac{s^2}{2}}\right)\times\left(e^{5s+\frac{s^2}{2}}\right)\times\left(0.95e^{3s+\frac{s^2}{2}}\right)\times\left(0.7e^{3s+s^2}\right)}{1-\left(0.05e^{6s+2s^2}\right)-\left(0.1e^{4s+s^2}\right)+\left(0.005e^{10s+3s^2}\right)} = P_{a-e} = w_{a-d(s)}\underline{s=0} \cong 0.777 \quad (27)$$

$$\frac{dM_{a-e(s)}}{ds} = \frac{dw_{a-e(s)}}{ds}\Big/0.777 \xrightarrow{s=0} \mu_{t:a-e} \xrightarrow{yields} \frac{10.7}{0.777} = 13.77 \quad (28)$$

$$\frac{dM_{a-e(m)}}{dm} = \frac{dw_{a-e(m)}}{dm}\Big/0.777 \xrightarrow{m=0} \mu_{c:a-e} \xrightarrow{yields} \frac{28.59}{0.777} = 36.79 \quad (29)$$

$$\frac{dM_{a-e(m)}{}^2}{d^2m} = \frac{dw_{a-e(m)}{}^2}{d^2m}\Big/0.777 \xrightarrow{m=0} 1063.5 \xrightarrow{yields} \frac{dM_{a-e(m)}{}^2}{d^2m} = \frac{1063.5}{0.777} = 1368.8 \quad (30)$$

$$s_{c(a-e)}{}^2 = 1368.8 - 1353.5 = 15.3 \xrightarrow{yields} s_{c(a-e)} = 3.91 \quad (31)$$

Considering the two existing loops in the figure with probabilities of 0.05 and 0.01, pieces with specified probabilities enter the loop through its nodes. Performance of iteration operation is repeated until this probability is deactivated for each piece. As to d-d and c-c loops, the possibility of the entrance of work-in-process piece to each loop is specified. The implementation condition for operation of loops is investigated considering the probability of each loop. If the work-in-process piece enters the loop repeatedly, the probability of its entrance tends to zero after three repetitions. Therefore, loops are opened up to three times and the network is shown in this way. It should be mentioned that the probability that each loop happens n times is lower than the probability of n-1 times occurrences of the same loop and this probability decreases after each reptilian. Through performing GERT calculations in the following loop-less network and its comparison with the calculations in the primary network, the difference was very insignificant and ignorable. It should be mentioned that time and cost calculations for each operation is performed in case of its occurrence with a specific probability. I.e. the entrance of the piece for retrieval operation (loop) should be probable in each repetition in order to be considered in time and cost calculations in that loop, therefore, considering these probabilities, the number of loop repetitions can be determined with a lower error. Performance of the loop opening operation in order to plot Gantt chart and investigation of some graphs for time and cost estimation in each phase of operation, are both applied in earned value calculations.

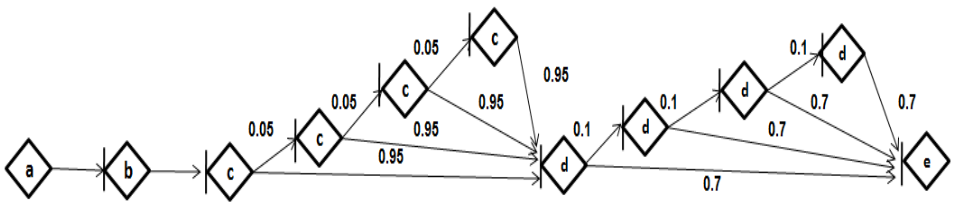


Fig.10. GERT network model with open loops

$$0.05^3 \times .95 \times .1^3 \times .7 = .000000083 \quad 0.05^2 \times .95 \times .1^3 \times .7 = .000001662 \quad 0.05 \times .95 \times .1^3 \times .7 = .00003325$$

$$0.05^3 \times .95 \times .1^2 \times .7 = .00000083 \quad 0.05^2 \times .95 \times .1^2 \times .7 = .00001662 \quad 0.05 \times .95 \times .1^2 \times .7 = .0003325$$

$$0.05^3 \times .95 \times .1^1 \times .7 = .0000083 \quad 0.05^2 \times .95 \times .1^1 \times .7 = .0001662 \quad 0.05 \times .95 \times .1^1 \times .7 = .003325$$

$$0.05^3 \times .95 \times .7 = .000083 \quad 0.05^2 \times .95 \times .7 = .001662 \quad 0.05 \times .95 \times .7 = .03325$$

$$\sum = .0000922 \quad \sum = .001846 \quad \sum = .03694$$

$$0.95 \times 0.1^3 \times 0.7 = 0.000665$$

$$0.95 \times 0.1^2 \times 0.7 = 0.00665$$

$$0.95 \times 0.1^1 \times 0.7 = 0.0665$$

$$0.95 \times 0.7 = .665$$

$$\sum = 0.738$$

$$0.0000992 + 0.001846 + 0.03694 + 0.738 = 0.777$$

$$p_{a-e} = \frac{0.95 \times .7}{1 - .05 - .1 + .005} = 0.777$$

As it can be observed, the total probabilities for network with and without opened nodes, obtained through Mason's rule, are almost equal. According to figure 10, the Mason's rule for open-loop condition (of the cost function) is shown below. Considering the cost average and variance calculations, the obtained values are almost equal for both conditions (presence of the loops and opening of the loops). Considering the figure above, the Gantt chart is plotted in probable conditions.

$$M_{a-e}(\text{no loop}) = \frac{.000000083e^{66m+15m^2} + .00000083e^{62m+13m^2} + .0000083e^{58m+11m^2} + .000083e^{54m+9m^2} + .000001662e^{60m+14m^2} + \cdots}{0.777}$$

$$\frac{+.00001662e^{56m+12m^2}+.0001662e^{52m+10m^2}+.001662e^{48m+8m^2}+.00003325e^{54m+13m^2}+.0003325e^{50m+11m^2}+.003325e^{46m+9m^2}+.03325e^{42m+7m^2}+\cdots}{\cdots}$$

$$\frac{.000665e^{48m+12m^2}+.00665e^{44m+10m^2}+.0665e^{40m+8m^2}+.665e^{36m+6m^2}}{\cdots}=\mu_{c:a-e}=\frac{dM_{a-e(m)}}{dm}\xrightarrow{m=0}36.79$$

$$\frac{dM_{a-e(m)}{}^{2}}{d^2m}\xrightarrow{c=0}\equiv 1368.8\ ,\ s^2=1368.8-1353.5=15.3\xrightarrow{yields}s_c=3.91$$

Average cost and standard deviation for the wholesystem

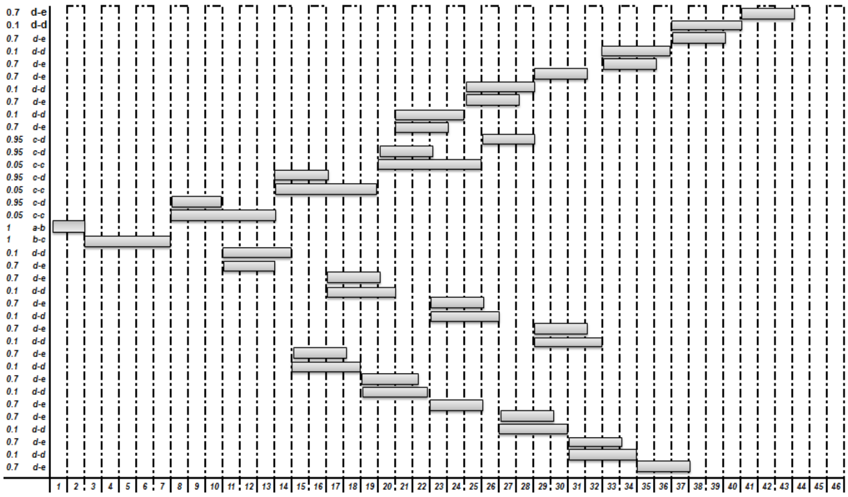


Fig.11.Gant chart of no loop network model

If we assume that the performance time is fixed for activities, in the average time spent for each activity would be considered as this certain time and considering the uncertain cost of each activity the system in investigated during the production at some moments of this process. Considering the probable Gantt plotted in figure 11, the approximate average of the production time is 43 days regardless of the number of the loop repetitions and the probability of each activity. Since the cost of the performance of each activity has a normal distribution function, the total percentage at investigation times is determined according to the average time of activity and as a cost percentage in the normal distribution function. The use of this method and determination of new average value and variance forms the moment-generating function and GERT value calculations up to the status date. Determination of three main factors (PV, EV, and AC) is considered here.

1. Time=0: $PV = 0, EV = 0, AC = 0$
2. Time=1:

During the investigation in the first day of the project, a-d activities are being performed. Through performing cost estimation and considering the proportion of investigation time to the total duration of activity, the average value and variance are calculated according to total average and variance of the activity and normality of executive cost. A new moment-generating function is developed in this way. After determination of moment-generating function the average value and variance would be calculated according to the Mason’s rule and the characteristics of generating functions. As it can be observed, loops are opened in order to investigate the project process and determine the earned value. Presence of loops complicates the analysis of earned value, there for existence of loop-less networks can be useful for analyzing the investigation dates.

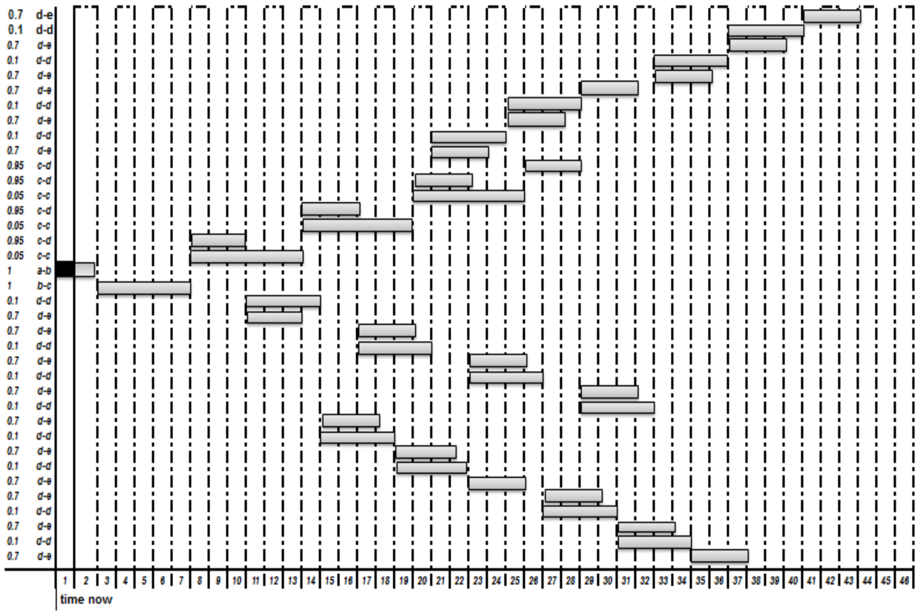


Fig.12.Gant chart with progress

$$activity_{(a-b)_{boudjet}} \rightarrow N\sim(7,2), l = 0.5 \rightarrow N\sim(3.5,0.5) \;,\; w_c = \left(e^{3.5m+0.25m^2}\right) \xrightarrow{yields} \frac{dM}{dm}(3.5+0.5m)\left(e^{3.5m+0.25m^2}\right) \xrightarrow{m=0} \mu_c = 3.5$$

$$\frac{dM_{(m)}{}^2}{d^2m} = \xrightarrow{m=0} 12.75 \;,\; s^2 = 12.75 - 12.25 = 0.5 \xrightarrow{yields} s_c \cong 0.7$$

$$f_c = \int_{\mu-b}^{\mu+b} \frac{1}{\partial\sqrt{2\pi}} e^{\frac{-(c-\mu)^2}{2\partial^2}} = 0.95 \rightarrow b = 1.4 = 2S \;,\; \mu_c \pm 2s = 3.5 \pm 1.4 = (2.1,4.9) \xrightarrow{yields} PV = (2.1,4.9)$$

The progress of a-b operation is about 32% up to the specified time.

$$activity_{(a-b)_{boudjet}} \rightarrow N\sim(7,2), l = 0.32 \rightarrow N\sim(2.24,0.2) \;,\; w_c = \left(e^{2.24m+0.1m^2}\right) \xrightarrow{yields} \frac{dM}{dm} = (2.24+0.2m)\left(e^{2.24m+0.1m^2}\right) \xrightarrow{m=0} \mu_c = 2.24$$

$$\frac{dM_{(m)}{}^2}{d^2m} = \xrightarrow{m=0} 5.217 \;,\quad s^2 = 5.217 - 5.07 = 0.2 \xrightarrow{yields} s_c \cong 0.45$$

$$f_c = \int_{\mu-b}^{\mu+b} \frac{1}{\partial\sqrt{2\pi}} e^{\frac{-(c-\mu)^2}{2\partial^2}} = 0.95 \rightarrow b = 0.9 = 2S \;,\; \mu_c \pm 2s = 2.24 \pm 0.9 = (1.346,3.134) \xrightarrow{yields} EV = (1.346,3.14)$$

$AC = 7.6$

3. At Time=2

$$w_c = \left(e^{7m+m^2}\right) \xrightarrow{yields} \frac{dM}{dm} = (7+2m)\left(e^{7m+m^2}\right) \xrightarrow{m=0} \mu_c = 7 \;,$$

$$\frac{dM_{(m)}{}^2}{d^2m} = \xrightarrow{m=0} 51 \;,\; s^2 = 51 - 49 = 2 \xrightarrow{yields} s_c = 2$$

$$f_c = \int_{\mu-b}^{\mu+b} \frac{1}{\partial\sqrt{2\pi}} e^{\frac{-(c-\mu)^2}{2\partial^2}} = 0.95 \rightarrow b = 4 = 2S \;,\; \mu_c \pm 2s = 7 \pm 4 = (3,11) \xrightarrow{yields} PV = (3,11)$$

The progress of a-b operation is about 65% up to the specified time.

$$w_c = \left(e^{4.55m+0.422m^2}\right) \xrightarrow{yields} \frac{dM}{dm}(4.55+0.845m)\left(e^{4.55m+0.422m^2}\right) \xrightarrow{m=0} \mu_c = 4.55 \;,\; \frac{dM_{(m)}{}^2}{d^2m} = \xrightarrow{m=0} 21.54$$

$$s^2 = 21.54 - 20.7 = 0.845 \xrightarrow{\text{yields}} s_c \cong 0.92$$

$$f_c = \int_{\mu-b}^{\mu+b} \frac{1}{\partial\sqrt{2\pi}} e^{\frac{-(c-\mu)^2}{2\partial^2}} = 0.95 \rightarrow b = 1.82 = 2S \text{ , } \mu_c \pm 2s = 4.55 \pm 1.82 = (2.73,6.37) \xrightarrow{\text{yields}} EV = (2.73,6.37)$$

$$AC = 16$$

4. At Time =4

Considering the average time of the predicted program, a-b operation has been finished at the end of the fourth date and two days have passed from the beginning of b-c operation. In order to investigate the cost and total moment-generating functions up to the investigated time, moment-generating functions for each operation have to be combined considering the predicted cost percentage up to that day.

$$M_Y(t) = M_{x_1}(t).M_{x_2}(t) \ldots M_{x_n}(t) = \left[e^{\mu_1 t + \partial_1^2 t^2/2}\right].\left[e^{\mu_2 t + \partial_2^2 t^2/2}\right] \ldots \left[e^{\mu_n t + \partial_n^2 t^2/2}\right] \rightarrow e^{[(\mu_1+\mu_2 \ldots \mu_n)t + (\partial_1^2 \ + \partial_2^2 \ \ldots + \partial_n^2 \ )t^2/2)]}$$

$$\text{activity}_{(b-c)_{\text{boudjet}}} \rightarrow N\sim(5,2), l = 0.4 \rightarrow N\sim(2,0.32) \text{ , } w_c = \left(e^{7m+m^2}\right).\left(e^{2m+0.32m^2}\right) = e^{9m+1.32m^2} \xrightarrow{\text{yields}} \frac{dw}{dm}(9 + 2.64m)\left(e^{9m+1.32m^2}\right) \xrightarrow{m=0} \mu_c = 9$$

$$\frac{dw_{(m)}{}^2}{d^2m} = \xrightarrow{m=0} = 83.64 \text{ , } s^2 = 83.64 - 81 = 2.64 \xrightarrow{\text{yields}} s_c = 1.63$$

$$f_c = \int_{\mu-b}^{\mu+b} \frac{1}{\partial\sqrt{2\pi}} e^{\frac{-(c-\mu)^2}{2\partial^2}} = 0.95 \rightarrow b = 3.26 = 2S \text{ , } \mu_c \pm 2s = 9 \pm 3.26 = (5.74,12.26) \xrightarrow{\text{yields}} PV = (5.74,12.26)$$

The actual progress of project up to the scheduled date is respectively equal to 100% and 15% for a-b and b-c operations.

$$w_c = \left(e^{7m+m^2}\right).\left(e^{0.75m+0.045m^2}\right) = e^{7.75m+1.045m^2} \xrightarrow{\text{yields}} \frac{dw}{dm} = (7.75 + 2.09m)\left(e^{7.75m+1.045m^2}\right) \xrightarrow{m=0} \mu_c = 7.75 \text{ , } \quad \frac{dw_{(m)}{}^2}{d^2m} = \xrightarrow{m=0} = 62.15$$

$$s^2 = 62.15 - 60.06 = 2.09 \xrightarrow{\text{yields}} s_c = 1.44 \text{ , } \mu_c \pm 2s = 7.75 \pm 2.88 = (5.74,12.26) \xrightarrow{\text{yields}} EV = (4.87,10.63)$$

$$AC = 20.2$$

5. Time=6

Considering the average time of the predicted program, a-b operation has been finished at the end of the sixth date and four days have passed from the beginning of b-c operation. In order to investigate the cost and total moment-generating functions up to the investigated time, moment-generating functions for each operation have to be combined considering the predicted cost percentage up to that day.

$$\text{activity}_{(b-c)_{\text{boudjet}}} \rightarrow N\sim(5,2), l = 0.8 \rightarrow N\sim(4,1.28)$$

$$w_c = \left(e^{7m+m^2}\right).\left(e^{4m+1.28m^2}\right) = e^{11m+2.28m^2} \xrightarrow{\text{yields}} \frac{dw}{dm} = (11 + 4.56m)\left(e^{11m+2.28m^2}\right) \xrightarrow{m=0} \mu_c = 11$$

$$\frac{dw_{(m)}{}^2}{d^2m} = \xrightarrow{m=0} = 125.56 \text{ , } s^2 = 125.56 - 121 = 4.56 \xrightarrow{\text{yields}} s_c = 2.13$$

$$f_c = \int_{\mu-b}^{\mu+b} \frac{1}{\delta\sqrt{2\pi}} e^{\frac{-(c-\mu)^2}{2\delta^2}} = 0.95 \rightarrow b = 4.27 = 2s \ , \ \mu_c \pm 2s = 11 \pm 3.264.27 = (6.73,15.27) \xrightarrow{\text{yields}} PV = (6.73,15.27)$$

The actual progress of project up to the scheduled date is respectively equal to 100% and 45% for a-b and b-c operations.

$$\text{activity}_{(b-c)_{\text{boudjet}}} \rightarrow N\sim(5,2), l = 0.45 \rightarrow N\sim(2.25,0.4)$$

$$w_c = \left(e^{7m+m^2}\right).\left(e^{2.25m+0.4m^2}\right) = e^{9.25m+1.4m^2} \xrightarrow{\text{yields}} \frac{dw}{dm} = (9.25+2.8m)\left(e^{9.25m+1.4m^2}\right) \xrightarrow{m=0} \mu_c = 9.25$$

$$\frac{dw_{(m)}{}^2}{d^2m} = \xrightarrow{m=0} = 88.36 \ ,$$

$$s^2 = 88.36 - 85.56 = 2.8 \xrightarrow{\text{yields}} s_c = 1.67 \ , \mu_c \pm 2s = 9.25 \pm 3.34 = (5.74,12.26) \xrightarrow{\text{yields}} EV = (5.91,12.59)$$

$$AC = 35$$

Considering the calculation of indicators at the status dates and the confidence interval of 95%, upper and lower limits were obtained for the planned value, earned value and actual cost. It should be mentioned that in calculation of earned value in certain condition, certain values would be obtained for the indicators and each of these indicators are identified under a graph, bur when there is a confidence interval; the analyses performed for each indicator on each date are introduced under an interval. Therefore, tow graphs are used for each indicator. Existence of an interval for each indicator excludes the obtained value from a certain condition and causes a risk in managers' decision making.

Table3: Numerical (cost) range in a confidence interval of 95% (normal)

Time	PV(Lower,Upper)	EV(Lower,Upper)	Ac
0	0	0	0
1	(2.1,4.9)	(1.34,3.14)	7.6
2	(3,11)	(2.73,6.37)	16
4	(5.74,12.26)	(4.87,10.63)	20.2
6	(6.73,15.27)	(5.91,12.59)	35

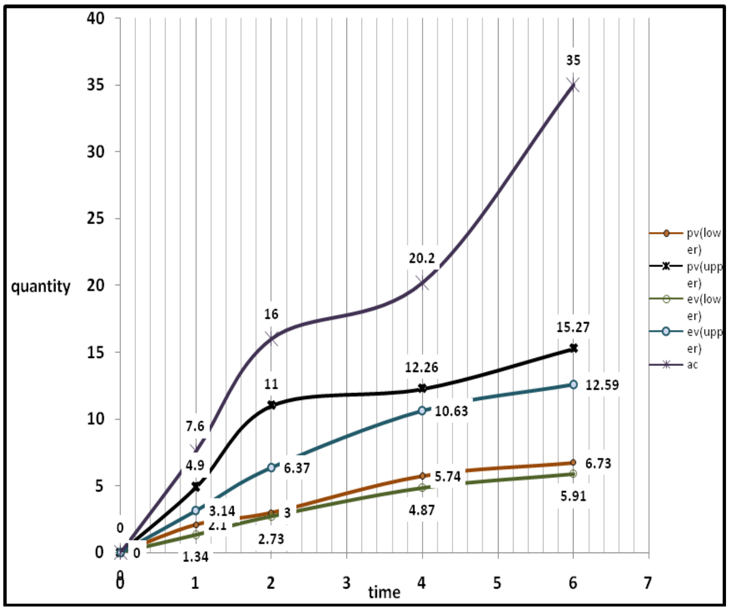


Fig.13.Upper and lower(PV,EV,AC) curves

Some scenarios were determined for cost and time performance indicators according to the performed calculations at the scheduled dates. These indicators apply upper and lower values foresaid calculations. According the upper and lower limits of the obtained values, the cost and time prediction process can be considered in for m of optimistic, most likely, and pessimistic scenarios. Considering the cost factor, the interval related to PV indicator approaches to pessimistic condition by being increased while a decrease in it value approximates it to optimistic condition. The interval related to EV indicator approaches optimistic condition trough being increased, while a decrease in its value approximates it to pessimistic condition. Considering the pessimistic point of view i.e. the lower limit of the earned value (EV) and upper limit of the planned Value (PV), the cost predictions are analyzed hereunder. It should be mentioned that as the difference in value between PV and EV increases, deviation from the schedule increases as well. The increase in difference between EV and AC would increase the deviation from the cost plan.

$$SPI=\frac{EV}{PV}\;,\;CPI=\frac{EV}{AC}\;,\;CPI_p=\frac{EV_l}{AC}\;,\;SPI_p=\frac{EV_l}{PV_U}\;,\;ETC=\frac{\mu_{BAC}-EV}{PF}\xrightarrow{yields}EAC_p=AC+ETC_p\;,PF_p=SPI_p\,.\,CPI_p$$

Table4: Predicted cost indicators and values in the pessimistic scenario

Time	CP	SP	PF	$\frac{\mu_{BAC}-EV}{PF_p}$	EA
0	….	….	….	….	….
1	0.18	0.27	0.04	886	893
2	0.17	0.25	0.04	851	867
4	0.24	0.4	0.1	319	339
6	0.17	0.39	0.07	441	476

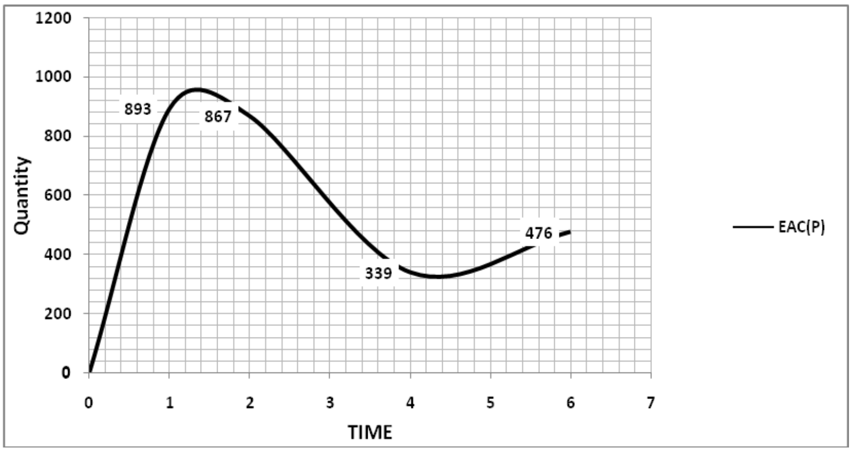


Fig.14.EAC(P) curve

Considering the data obtained through cost prediction calculations and since the time and cost values are defined under (normal) distribution functions) with a specific standard deviation, the cost values vary from optimistic to pessimistic in the numerical range. In operation cost prediction table, the lower limit of EV and The upper limit of PV have been applied in order to calculate and plot the indicators in pessimistic condition. The obtained graph can be used for cost prediction in order to help the conservative managers to make decisions. As it has been proved, determination of a certain number to predict the cost is meaningless, which increases the risk of decisions.

7. Comparative analysis:

Three elements of earned value management are excluded from the certain conditions through performing calculations using distribution functions and determining the average value and variance of time and cost (specially the cost). The existence of uncertainty and determination of numerical ranges in order to predict the production cost, refers to the significance of managers' viewpoint in making right decisions, therefore, the budget prediction varies depending to the ideas of decision makers that ranges from conservative to optimistic. According to the traditional attitudes toward earned value management, the values of time and cost are certain and they are investigated in form of average values. Therefore, budget prediction is completely certain and no pessimistic or optimistic viewpoints are issued here. Considering the tables below, the certain values are calculated in probable condition (average) which results in prediction of cost in average condition. Considering the existing risk in the projects and the occurred swings during their performance, there would be general changes in the primary program. Leading and controlling this condition (particularly about the cost) varies considering the managers' viewpoints and their art of management in such situations.

Table5: Three EVM elements in uncertain condition

Time	pv(deterministic)	ev(deterministic)	Ac
0	0	0	0
1	3.5	2.24	7.6
2	7	4.55	16
4	9	7.75	20.2
6	11	9.25	35

Table6:Three EVM elements in certain condition

Time	pv(Lower,Upper)	ev(Lower,Upper)	Ac
0	0	0	0
1	(2.1,4.9)	(1.34,3.14)	7.6
2	(3,11)	(2.73,6.37)	16
4	(5.74,12.26)	(4.87,10.63)	20.2
6	(6.73,15.27)	(5.91,12.59)	35

8. Conclusion

The time and cost of production operations are investigated through GERT network. Since these variables are discussed in probable condition in form of distribution functions, judging them in certain condition cause an error in managers' decision making process. The results obtained through probability analysis of time and cost in the network contained loops and Mason's rule was applied in order to coordinate this network with its loop-less equivalent. Since each loop indicates the repetition of its operation considering the probability of its performance, a loop-less network with more routes is developed through opening each loop and creating a right number of direct routes. Existence of probability factors of loop occurrence is very important in this coordination, because each loop stops its performance after a specific number of repetitions. Due to the developments in production process and probable predictions of time and cost, Earned Value Management is considered as a method that can be applied in order to predict time and cost. In such probable condition, considering the existence of time and cost numerical intervals for EV elements, the indicators of this method are placed under a variable circumstance. According to the investigation results, even the predicted values of total cost have significant variations at investigated dates. Therefore, certain statement of total cost in probable condition would cause an error in programming. Considering the intervals determined for EV indicators, decision making condition varies depending on managers who might or might not accept the risk.

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