A Decision Making Model for Project Portfolio Selection in Uncertainty Environment

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Abstract: Project portfolio selection for making decisions on investment is a critical decision in such companies. This selection is a multi-criteria problem, due to miscellaneous criteria which are often in conflicting with each other. The researchers define four criteria for selecting the optimal project portfolio as cost, time, scope and risk criteria. On the other, project portfolio selection problem is often influenced by uncertainty in practice; also because of the uncertainty associated with imprecision, loss of information and lack of understanding. In this paper, the implementation of an organized framework for project portfolio selection has discussed through the proposed model base on fuzzy TOPSIS approach using linguistic terms in order to calculate the importance holistic weights of evaluation criteria and rank the feasible project portfolios in descending order. To show the potential applications of the methodology, the proposed method is used for a real world case study.

1. Introduction

The projects have a profound impact on the modern organization. Understanding the role of projects in achieving organization's strategic goals mainly increased in recent years. However, still the wastage of resources through improper selection of projects or their improper formulation is immense. Together these two factors limit the growth potential of the organization and undermine its competitive position. Trying to answer this challenge, one can use techniques proposed in the project portfolio management. Although it is sometimes recognized as another project management methodology, in fact it is something else. Project portfolio management goes beyond project management. It links the organization's vision and its strategic goals with the process of project selection, their implementation and consumption of their benefits. The key to a new way of project portfolio life cycle management is to select the right projects at the right time.

It is noted that diversity of the portfolio should be analyzed before the final composition of the portfolio is determined. This can be achieved by including projects that differ in type, size and risk.

Each modern organization must implement various projects. While managers cope better and better with individual projects, managing multiple projects still poses quite serious problems.

Let us start with defining basic concepts, such as program, project portfolio, and project portfolio management. Wysocki and McGary [1] define a program as a set of projects that must be done in a specific order. According to them, a portfolio is a collection of projects that are closely connected in some way. In this approach, the portfolio is considered to be a more general term than a program. While the condition for program completion is the realization of a sequence of projects, and the program itself, like a project, has a specific purpose, cost, budget, and completion date, a portfolio is constructed continuously.

On the other hand, a project portfolio is a group of projects or programs in an organization or business unit that purposes at strategic goals, share resources, and must compete for funding.

According to the methodology proposed by the Project Management Institute (PMI) program is a set of related projects managed in a coordinated way to obtain benefits and control not achievable from managing them individually [2]. It is pointed out that in contrast with project management; program management is the centralized, coordinated management of a group of projects to achieve the program's strategic objectives and benefits. In PMBOK a portfolio is defined as a collection of projects or programs and other works that

are grouped together to simplify effective management of that work to meet strategic business objectives. It is noted that the projects or programs in the portfolio does not have to be interdependent or directly related.

The portfolio construction depends on the targets to be achieved by managing portfolio. PMBOK states that one goal of portfolio management is to maximize the value of portfolio. This can be achieved by careful examination of candidate projects and programs for inclusion in the portfolio and timely exclusion of projects not meeting the portfolio's strategic objectives. Senior managers or senior management teams responsible for portfolio management should ensure the right balance among incremental and radical finances and the efficient use of resources.

Defining project portfolio management Wysocki and McGary [1] say that it includes five main tasks:

• formulating investment strategy of the portfolio;
• specifying types of projects eligible for the portfolio;
• evaluating and prioritizing projects that are candidates for the portfolio;
• constructing a balanced portfolio that meets the investment objectives;
• The monitor of the portfolio implementation and adjusting the composition of the portfolio in order to achieve the desired results.

Gray and Larson [3] emphasize that every organization should build its own project portfolio management system. Defining the rules for the allocation of resources is also extremely important.

Portfolio selection is concerned with selecting a combinationofsecuritiesamongportfolioscontaininglargenumbersofsecuritiestoreachtheinvestment goal.

In the proposed portfolio selection models, security returns are assumed to be random variables, and random uncertainty is considered as the sole way of modeling uncertainty. In real world, there are many non probability factors that affect the stock markets and they should not be dealt with probability approaches. With the introduction of fuzzy set theory and possibility theory [4-6], several scholars began to employ this theory to manage portfolios in a fuzzy environment. For example, Tanaka et al. [7] and Inuiguchi and Tanino [8] assumed the

security returns to be fuzzy variables with possibility distributions and proposed the possibility portfolio selection models, respectively, Parra et al.[9] proposed a fuzzy goal programming approach for portfolio selection, and Zhang and Nie [10] proposed the admissible efficient portfolio model.

Most of the existing multi-period portfolio selection models have focused on only two fundamental factors, i.e., expected return and risk of portfolio. However, in practical investment, only using expected return and risk as decision criteria cannot capture all the relative information for a portfolio decision.To provide investors with additional choice, more criteria shouldbe integrated into portfolio selection models. To our knowledge, studies that have considered multiple decision making criteria arefew for the multi-period portfolio selection problem.

Up to now,the mainly existing portfolio selection models are based on probability theory or fuzzy set theory, therefore only one kind of uncertainty, randomness or fuzziness is reflected.In fact, randomness and fuzziness are often mixed up together in the real setting which requires taking into account them simultaneously in portfolio selection process.

Fuzzy random variable (f.r.v) introduced by Kwakernaak [11] in 1978 and random fuzzy variable introduced by Liu [12] in 2002 are appropriate ways to describe the uncertainty of randomness and fuzziness required to be considered simultaneously. Roughly speaking, a f.r.v is a measurable function from a probability space to a collection of fuzzy sets, and a random fuzzy variable is a function from a possibility space to a collection of random variables.

Recently, Katagiri and Ishii [13] considered an application of f.r.v to a single index model, and proposed a portfolio selection model to deal with the randomness and fuzziness simultaneously based on possibility theory and a chance constrained model in stochastic programming. Huang [14] employed the random fuzzy theory [15,16] to study portfolio selection in a random fuzzy environment in which the security returns are assumed to be stochastic variables with fuzzy information.

Since stock experts possess enough information and experience, it is a good method to let them provide their rough estimation about the future returns of securities.

In this paper, following the idea of mean variance model, we propose a new portfolio selection model which combines the statistical techniques with the experts' judgment based on fuzzy random theory.

2. Material & Methods
2.1 Fuzzy TOPSIS approach with subjective and objective weights
Technique for Order Performance by Similarity to Ideal Solution (briefly called TOPSIS) is one of the most important techniques among MCDM methods.TOPSIS bases upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution (maximum benefit criteria and minimum cost criteria) and the farthest from the negative ideal solution (minimum benefit criteria and maximum cost criteria).It was developed by Chinese researchers.(Hwang and Yoon -1981)

2.1.1. Subjective and objective weight

Basically in MCDM approaches weights of features play the main role in every decision making process. Since the evaluation of criteria contains various thoughts and views, we can't consider that their importance is the same. (Chen, Tzeng, & Ding, 2003).The two main classification of weighting methods are subjective methods and objective methods. The subjective methods are just based on priorities of decision makers. It is calculated for overall evaluation of each decision maker by using mathematical methods like: eigenvector method, weighted least square method, and mathematical programming models. The objective methods specify weights automatically by solving mathematical models regardless of the decision maker's judgments. Methods like entropy, multiple objective programming, etc.
To improve affecting of evaluation results by the weighting approaches both of them (subjective and objective methods) were used in the research. As previously mentioned subjective weighting is according to decision maker's perspectives and experiences while the objective one relies on mathematical calculations. Basically where we can't get trustworthy subjective weights it is proper to apply objective weights (Deng, Yeh, & Willis, 2000).

Researchers had offered many objective weighting measures. The Shannon's concept in entropy (Shannon & Weaver, 1947) is one of them. According to his opinion proper information about the characteristics involved in an objective weighting measurement transmitted to the decision makers (Zeleny, 1996).
A measuring tool for determining the uncertainty in information formulated in terms of probability theory is Shannon entropy concept. Three features for a measure of information in communication stream were extended by Shannon. The next studies used his results to an extensive variety of applications like spectral analysis (Burg, 1967), language modeling (Rosenfeld,1994) and economics (Golan, Judge, & Miller,1996).
He extended his measure (H) which met the following features for all pi within the estimated joint probability distribution (P) (Zitnick&Kanade, 2004):
1. H is a continuous positive function.
2. If all pi are equal,$p_i = \frac{1}{n}$,then H should be a monotonic increasing function of n.
3. For all$n \geq 2, H(p_1, \cdots, p_n) = H(p_1 + p2, p3 \cdots, pn + p1 + p2H (p1p1 + p2 , \frac{p2}{p1+p2})$.

Shannon showed the only function that satisfied these propertiesis
$$H(P) = -\sum_i p_i \log (p_i) \quad (1)$$

We develop using the concept of Shannon entropy as a measure of weight calculation method. Entropy weight is a parameter that explains the relationship between various options toward a specific feature. Whatever higher entropy value, entropy weight and the diversity of options will be smaller in thisspecific feature and finally less information will be provided by that feature and consequently it makes less important of the feature in decisionmaking process.

2.1.2. Subjective and objective weight

In this paper, we offer a developed TOPSIS method which integrates subjective and objective weights. Using this method has double advantages. The first is to benefit of the expertise of decision makers and the second

one is involving the end-users in whole process of decision-making. In addition decision makers assigned the subjective weights, theShannon's entropy is usedas a foundationfor computingthe subjective weights. We accepted the concept of information entropy to approve the weight of evaluating feature that can effectively balance the effect of subjective factors. The innovative approach can provide a more general approach to the decision making process.

The steps of fuzzy TOPSIS algorithm can be expressed asfollows:

Step 1: Construct a decision matrix

Assume there *m* alternatives (software products) $A_i (i = 1, 2, …, m)$ to be evaluated against n selection criteria $C_j (j = 1, 2, …, n)$ Subjective assessments are to be made by DM to determine (a) the weighting vector $W = (w_1, w_2, \cdots, w_j, \cdots, w_n)$ and (b) the decision matrix $X = \{x_{ij}, i = 1,2 \cdots, m; j = 1,2 \cdots, n$, using the linguistic terms given in Table 1. The weighting vector W represents the relative importance of n selection criteria $C_j (j = 1, 2, …, n)$ for the problem. The decision matrix $X = \{x_{ij}, i = 1,2 \cdots, m; j = 1,2 \cdots, n\}$, represents the utility ratings of alternative A_i with respect to selection criteria C_j. Given the weighting vector W and decision matrix X, the objective of the problem is to rank all the alternatives by giving each of them an overall utility with respect to all selection criteria. The decision matrix can be expressed as follows:

$$D = \begin{matrix} & C_1 & C_2 & …C_n \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & X_{2n} \\ \vdots & & \ddots & \vdots \\ X_{m1} & X_{m2} & \cdots & X_{mn} \end{bmatrix}$$

$$W = [\, w_1 \quad w_2 \quad \cdots \quad w_n]$$

Step 2: In this step, we both use subjective weighting method and entropy-based objective weighting method.

(a) Subjective: Determine the DM's weights for each criterion

$$\widetilde{W}_j = \frac{1}{n}\left(\sum_{j=1}^{n} w_j^e\right), \quad j = 1, 2, …. n$$
(2-1)

(b) Objective: In order to determine objective weights by the entropy measure, the decision matrix needs to be normalized for each criterion $c_j (j = 1, 2, …, n)$ to obtain the projection value of each criterion: p_{ij}.

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad (2\text{-}2)$$

After normalized the decision matrix, we can calculate the entropy values e_j as

$$e_{j = -k} \sum_{j=1}^{n} p_{ij} \ln p_{ij} \qquad (2\text{-}3)$$

k is a constant, let $k = (\ln(m))^{-1}$ The degree of divergence di of the intrinsic information of each criterion $C_j (j = 1, 2, …, n$ may be calculated as

$$d_j = 1 - e_j \qquad (2\text{-}4)$$

The value d_j represents the inherent contrast intensity of c_j. The higher the d_j is, the more important the criterion c_j is for the problem. The objective weight for each criterion can be obtained.

$$W_j = \frac{d_j}{\sum_{k=1}^{n} d_k} \qquad (2\text{-}5)$$

Step 3: Calculate the aggregate weights for each criterion W_j as follows:

$$\widetilde{X}_{ij} = \frac{1}{n}\left(\sum_{e=1}^{n} \tilde{x}_{ij}^e\right), \quad i = 1, 2, …, m \quad (3)$$

Step 4: Obtain the decision matrix to identify the *j*th criteria with respect to *i*th alternative.

$$\widetilde{R} = [\tilde{r}_{ij}]_{m \times k} \qquad (4)$$

Step 5: Normalize the decision matrix in order to make each criterion value is limited between 0 and 1, so that each criterion is comparable. The initial data

with respect to each criterion will be normalized by dividing the sum of criterion values. For fuzzy data denoted by triangular fuzzy number as (a_{ij}, b_{ij}, c_{ij}) ,the normalized values for benefit-related criteria and cost-related criteria are calculated as follows:

$$\tilde{r}_{ij} = \left\{ \left(\frac{a_{ij}}{c_{ij}^+}, \frac{b_{ij}}{c_{ij}^+}, \frac{c_{ij}}{c_{ij}^+} \right) j \in B \right. , \qquad (5\text{-}1)$$

$$\tilde{r}_{ij} = \left\{ \left(\frac{a_{ij}}{c_{ij}^-}, \frac{b_{ij}}{c_{ij}^-}, \frac{c_{ij}}{c_{ij}^-} \right), j \in C \right. \qquad (5\text{-}2)$$

[

$$c_j^+ = \max_i c_{ij} \, if \, j \in B$$
$$a_j^- = \min_i a_{ij} \, if \, j \in C$$

Step 6: Calculate the overall accomplishment evaluation for each alternative by multiplying the aggregate weights for each normalized criterion

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times k} , i = 1, 2, ..., m \qquad j = 1, 2, ..., n \quad (6\text{-}1)$$
$$\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_{ij} \qquad (6\text{-}2)$$

Step 7: Determine the positive ideal solution A^+ and the negative ideal solution A^-. Sort the weighted normalized values for each criterion in descending order.

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \cdots, \tilde{v}_k^+) \qquad (7\text{-}1)$$
$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \cdots, \tilde{v}_k^-) \qquad (7\text{-}2)$$

Step 8: Calculate the distance from the positive ideal solution & the negative ideal solution for each alternative. According to Bojadziev and Bojadziev (1995), the distance between two triangular fuzzy numbers $A_1 = (a_1, b_1, c_1)$ and $A_2 = (a_2, b_2, c_2)$ is calculated as

$$d(A_1, A_2) = \sqrt{\frac{1}{3} \left[(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 \right]}$$
$$\qquad (8)$$

$$d_i^+ = \sum_{j=1}^k d(\tilde{v}_{ij}, \tilde{v}_j^+) , i = 1, 2, ..., m$$
$$d_i^- = \sum_{j=1}^k d(\tilde{v}_{ij}, \tilde{v}_j^-) , i = 1, 2, ..., m$$

Step 9: Calculate the closeness coefficient (CC). And rank each CC of each alternative in descending order. The alternative with the highest CC value will be the best choice.

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} , i = 1, 2, ..., m$$
$$\qquad (9)$$

2.2 Experimental result

In this section, an example is provided. A project selection problem can be calculated as a multiple criteria decision making problem in which alternatives are the projects to be selected and criteria are those attributes under consideration. A company desires to select a new project portfolio in order to improve company productivity. After preliminary screening, four alternatives A_1, A_2, A_3, A_4 have remained in the candidate list. Four experts, D_1, D_2, D_3, D_4 form a committee to act as decision makers. There are four criteria need to be considered:

(1)Costs of each project portfolio investment (C_1).
(2)Time of completing of each project portfolio (C_2).
(3) Scope of work of each project portfolio (C_3).
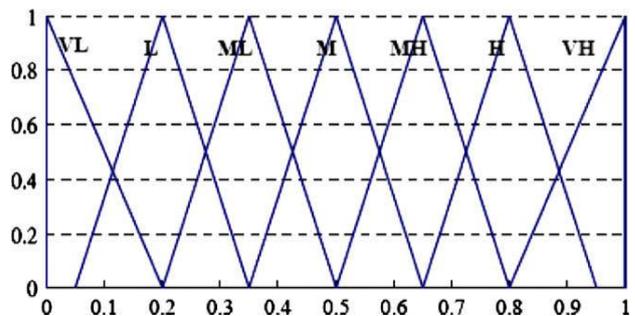(4) Risk of each project portfolio (C_4).

Fig. 1.The fuzzy linguistic variables for each criterion.

Table 1

Fuzzy linguistic terms and correspondent fuzzy
numbers for each criterion.

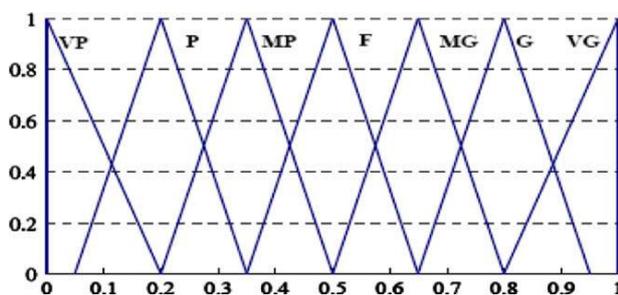| importance	Abbreviation	Fuzzy Number
Very Low	VL	(0,0,0.2)
Low	L	(0.05,0.2,0.35)
Medium low	ML	0.2,0.35,0.5)
Medium	M	(0.35,0.5,0.65)
Medium high	MH	(0.5,0.65,0.8)
High	H	(0.65,0.8,0.95)
Very high	VH	(0.1,1)

Fig. 2.The fuzzy linguistic variables for each
alternative.

Table 2

Fuzzy linguistic terms and correspondent fuzzy
numbers for each alternative

importance	Abbreviation	Fuzzy Number
Very poor	VP	(0,0,0.2)
Poor	P	(0.05,0.2,0.35)
Medium poor	MP	(0.2,0.35,0.5)
Fair	F	(0.35,0.5,0.65)
Medium good	MG	(0.5,0.65,0.8)
Good	G	(0.65,0.8,0.95)
Very good	VG	(0.8,1,1)

Table 3

Each criterion weight in linguistic term.

C	DM			
	D_1	D_2	D_3	D_4
C_1	H	VH	H	VH
C_2	VH	ML	VH	H
C_3	MH	VH	M	H
C_4	ML	L	MH	VH

Table 4

Each criterion weight fuzzy number.

Criterion	Fuzzy number
C_1	(0.725,0.900,0.975)
C_2	(0.613,0.788,0.863)
C_3	(0.575,0.738,0.850)
C_4	(0.388,0.550,0.663)

Table 5

Each criterion projection value.

C	DM			
	D_1	D_2	D_3	D_4
C_1	0.83	0.92	0.78	0.94
C_2	0.92	0.35	0.93	0.74
C_3	0.62	0.90	0.45	0.72
C_4	0.31	0.24	0.68	0.93

Table 6

Entropy-based weights.

C	e_j	d_j	w_j
C_1	0.3486	0.6513	0.3917
C_2	0.5298	0.4702	0.2828
C_3	0.7120	0.2880	0.1730
C_4	0.7468	0.2531	0.1522

Table 7

The initial DM rating table.

	C$_1$				C$_2$				C$_3$				C$_4$			
	D$_1$	D$_2$	D$_3$	D$_4$	D$_1$	D$_2$	D$_3$	D$_4$	D$_1$	D$_2$	D$_3$	D$_4$	D$_1$	D$_2$	D$_3$	D$_4$
A$_1$	F	G	G	V	MP	F	F	F	G	F	F	G	F	MG	MG	P
A$_2$	G	G	G	F	VG	F	P	F	P	F	P	F	F	P	VP	P
A$_3$	G	F	VG	VG	G	F	F	G	G	MG	G	F	G	F	F	G
A$_4$	MG	G	G	G	F	F	G	VG	G	MG	VG	G	G	G	G	F

Table 8
Normalized decision matrix.

	C$_1$	C$_2$	C$_3$	C$_4$
A$_1$	(0.671,0.849.0.973)	(0.385,0.569,0.754)	(0.541,0.703,0.865)	(0.400,0.571,0.743)
A$_2$	(0.630,0.795,0.959)	(0.477,0.677,0.815)	(0.216,0.378,0.541)	(0.129,0.257,0.443)
A$_3$	(0.712,0.904,0.986)	(0.615,0.800,0.985)	(0.500,0.662,0.824)	(0.571,0.473,0.914)
A$_4$	(0.671,0.836,1.000)	(0.662,0.862,1.000)	(0.703,0.878,1.000)	(0.657,0.829,1.000)

Table 9
Overall rating for each alternative.

	C$_1$	C$_2$	C$_3$	C$_4$
A$_1$	(0.386,0.626.0.827)	(0.149,0.313,0.499)	(0.392,0.632,0.843)	(0.245,0.450,0.641)
A$_2$	(0.362,0.586,0.815)	(0.185,0.372,0.540)	(0.157,0.341,0.527)	(0.079,0.203,0.382)
A$_3$	(0.410,0.667,0.838)	(0.238,0.440,0.652)	(0.363,0.596,0.804)	(0.350,0.585,0.789)
A$_4$	(0.386,0.616,0.850)	(0.256,0.474,0.663)	(0.509,0.791,0.975)	(0.403,0.655,0.863)

Table 10
Closeness coefficient table.

	d$^+$	d$^-$	cc	Ranking
A$_1$	2.619	2.676	0.505	3
A$_2$	3.138	2.119	0.403	4
A$_3$	2.350	2.963	0.558	2
A$_4$	2.135	3.224	0.602	1

The proposed fuzzy TOPSIS method is applied to solve this problem, and the computational procedure is summarized as follows:

Step 1: The research uses the linguistic variables developed by Chen and Hwang (1992) as Fig. 1 and Table 1 for each criterion. We use triangular fuzzy number to express importance of each criterion. The linguistic

terms range from "very low" to "very high". The specific term "very" is utilized to stress the degree of each criterion.

Step 2: Same as Step 1. Determine each alternative's linguistic term and fuzzy number in Fig. 2 and Table 2. We use triangular fuzzy number to express evaluation of each alternative. The linguistic terms range from "very poor" to "very good". The Table 2 illustrates each fuzzy linguistic term to its correspondent fuzzy numbers for each alternative.

Step 3: Each DM may rate each criterion's weight with respectto linguistic term. That means one expert may apply his/her own expertise to judge how important a criterion is. The resultis shown in Table 3.

Step 4: Refer to (2), the aggregated fuzzy rating and fuzzy weight of each criterion are shown in Table 4.
Besides subjective weights, we apply entropy method to calculate objective weight for each criterion.According to Table 3, wederive its crisp projection for each criterion in Table 5.
According to (1-2) to (1-4), we calculate e_j, d_j and w_j respectively. The Table 5 shows the entropy-based weighting results.From above, we may clearly identify that C_1 is the mostimportant criterion.

Step 5: Each DM rates each alternative with respect to each criterion.Since the judgments would be partially depends on personalpreference, DM'srecommendation is applied fuzzylinguistic terms. By applying (2‐4), the original DM rating tableand normalized fuzzy decision matrix are shown in Tables 7and 8 respectively.

Step 6: According to (5),calculating each alternative's overallrating and is shown in Table 9.

Step 7: According to (6) and (7), we calculate each alternative'sPIS and NIS. Then calculate the closeness coefficient (CC) anddetermine the best alternative. The result is shown in Table 10.As the result from Table 10, the order of rating among thosealternatives is:

$A_4 > A_3 > A_1 > A_2$, The best alternative wouldbe A_4.

4. Results
The project portfolio selection process is a methodology for evaluating and ranking the feasible project portfolio and selecting the best alternative using a multi-criteria decision making technique with the help of linguistic terms can be very useful to take into account uncertainty. For achieving the purpose, the TOPSIS method under fuzzy environment was employed in order to obtain the importance of portfolio. A fuzzy modification of the TOPSIS method based on subjective and objective weights is applied to calculate weights of criteria and overall rating scores of the alternatives.[17] The proposed method is employed for a portfolio selection problem. The results show that the risk of project portfolio is selected as the optimal criteria for selecting the project portfolios.

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