Third Order NLM Filter for Poisson Noise Removal from Medical Images

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Abstract

Poisson noise is a signal dependent noise for which a number of filters have been used in the literature including Non Local Mean (NLM) filter. In this work we are proposing a modified NLM filter consisting of three variants based on the fact that Third Order moment of the image makes the signal and the noise more outstanding and prominent and thus making it easy to be de-noised. Proposed method is applied on general image Lena as well as on medical images for reconstructing the original image contaminated by non-Gaussian noise and the results are compared with standard standalone NLM filter visually and using statistical indices including the correlation. Results show that the proposed variants are performing more efficiently when compared to conventional filters.

Keywords: Poisson noise, Noise removal, Medical image de-noising, Medical image processing

1. Introduction

Medical imaging plays an important role in disease diagnostics as well as pre-operative surgical procedures. Such medical imaging becomes viable solution for many problems and healthcare challenges in developing countries. For examining the internal parts of body without opening it, images are acquired with different medical imaging modalities. Images processed offline become a source of sharing, viewing and assessing many complications that can become useful in tele-consultation of patients, and containing some diseases from reaching outbreaks. Medical imaging is consisting of technological components interfaced to image modalities and application system all linked together on computer networks for acquisition purposes. However, there are serious challenges mainly due to the physical conditions these images are acquired in, as such images may become worse by getting coupled with diverse sorts of noises. To mention few, Magnetic Resonance Imaging (MRI) is associated with Rician noise, Ultrasound images by speckle noise, Single Photon Emission Computed Tomography (SPECT) and Positron Emission Tomography (PET) by Poisson noise [1, 2, 3, 4].

In the domain of medical imaging, noise removal is considered as one of the essential pre-processing steps. Because of the Additive nature, Additive White Gaussian Noise (AWGN) is very easy to be removed from images and that is why a large number of de-noising algorithms for the purpose are developed. On the other hand removal of non-Gaussian noise is very challenging due to its signal dependency and multiplicative nature [5]. Rician noise and Poisson noise are examples of non-Gaussian noise, which easily get associated in different medical imaging modalities. The general mathematical model for multiplicative and signal dependent noise [6] is given as (1)

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$$g(u,v) = s(u,v) * h(u,v) + n(u,v) \quad (1)$$

Where $g(u,v)$, $h(u,v)$, $n(u,v)$ and $s(u,v)$ are defined as noisy images, blurring kernel, noise and original image (which is needed to be recovered) respectively. For recovery of the original image, blurring kernel is dropped and model of degradation is given as (2)

$$g(u,v) = s(u,v) + n(u,v) \quad (2)$$

Whereas, the mathematical model for Multiplicative noise is given as

$$g(u,v) = s(u,v).n(u,v) \quad (3)$$

The magnitude of Poisson noise varies across the image, as being of signal dependent nature, it varies with the image intensity, making thus removing of such noise more difficult. In this paper we focus on the variants form of NLM filter, which when applied on a general Lena image, is making the signal and noise outstanding for easy de-noising. The obtained results compared with NLM standalone filter in terms of statistical indices beside general visual appearance.

2. Poisson Noise

Poisson noise is an electronic noise that occurs when some limited number of particles, for example, electrons in an electronic circuit (or photons in optical devices),is holding energy which is little enough to turn out some measurable statistical variations in evaluation. Poisson noise is also known by photon or short noise.

Poisson Noise is a multiplicative noise having an unsure random nature associated with the estimation of light. The occurrence of Poisson noise is signal dependent and is conceived as a high cause of image noise, however, not under low light conditions.

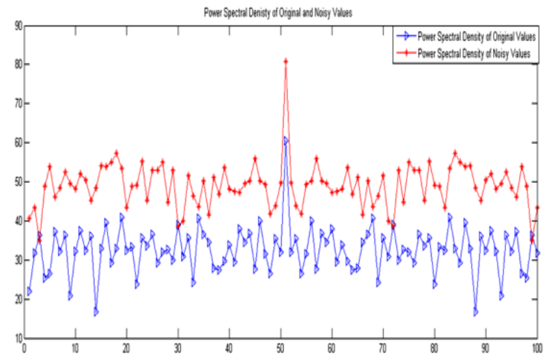

Figure 1: Poisson Noise Signal Dependency by Power Spectral Density

For showing the signal dependence of the Poisson noise the matrix of 10x10 order of random values is taken and power spectral density of it is taken shown in figure 1. Poisson noise is added in matrix and power spectral density is taken. The plot clearly shows that the nature of Poisson noise is signal dependent.

Let the original image is denoted by $m = \{m_{i,j} : i,j = 1, \dots \dots \dots N\}$ and noisy image by $n = \{n_{i,j} : i,j = 1, \dots \dots \dots N\}$, whereas the values of noisy image are contaminated by Poisson noise. So for a given true image m, the likelihood for observing n is

$$p(n/m) = \prod_{i,j=1}^{N} \frac{e^{-m_i} m_{i,j}^{n_{i,j}}}{n_{i,j}!} \quad (4)$$

The approach adopted in this work depends on the Non Local Mean for noise removal with a modified higher order noisy image.

3. Non Local Mean

Most of de-noising algorithms are designed by having two assumptions about the image [7]. These assumptions lead to the loss of fine details and blurring the de-noised image. The first assumption is about original image, it does not hold fine details, that is, it only consists of low frequencies. The second assumption is about

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noisy image, it holds both low and high frequencies. The noise is considered non-smooth, as it has high frequencies. The Wiener and Gaussian approaches follow these assumptions as well [7]. While de-noising the noisy image they remove higher frequencies from noisy image leaving behind the low frequencies. However, some images have structure and fine details with higher frequencies. As such these methods are following the above assumptions so they cannot differentiate between higher frequencies of noisy image and original image. Therefore, such techniques result in blurring image due to loss of higher frequency contents in the original image. To prevent such things from happening Buades [8] proposed Non Local Means (NLM) filter. This approach follows the similarity concept instead of the above stated assumption resulting into losing the high frequency components.

The Non Local Mean filter [9, 10] is modified form of Yaroslavsky filter [11], which averages out the pixels having intensity of similarity locally. The major difference between Non Local Means and Yaroslavsky filter is that similarity between pixels by using the concept of region comparison rather than the pixel comparison. This approach, which is based on matching and is not limited to local pixels is more effective in noise removal. This technique is particularly useful in de-noising low signal-to-noise-ratio (SNR) images where the intensity gradient due to noise elements may compete with or even exceed the intensity gradient due to features in the images. Figure 2 shows the self similarity concept used in NLM filter.

Figure 2: Self Similarity Concept in an Image

When comparing the pixels R, S1, S2 and S3, the pixel S1 and S2 have higher weight because of similarity windows are similar to window of R. Whereas the S3 have very small weight because intensity grey scale values in similarity windows are too different.

For a given image $U(j)$ the filtered value at any point $i$can be computed by using NL-means as weighted average of neighboring pixels $\aleph_i$ in image [12] by using following equation (5)

$$NL(U(i)) = \sum_{j\epsilon\aleph_i} w(i,j)U(j) \tag{5}$$

With $0 \leq w(i,j) \geq 1$ and $\sum_{j\epsilon\aleph_i} w(i,j) = 1$, where i is point being filtered and j represents any other pixel. The weight $w(i,j)$ is based on similarity between neighborhoods $\aleph_i$ and $\aleph_j$ of pixels i and j. However, $\aleph_i$ is defined as the square neighborhood window centered at pixel$i$. The weight $w(i,j)$ in NLM is calculated as

$$w(i,j) = \frac{1}{W(i)} \exp\left({}^{-d(i,j)}/_{h^2}\right) \tag{6}$$

Where h defines the filtering parameter and $d(i,j)$ define the Gaussian weighted Euclidean distance calculated as under [13].

$$d(i,j) = \left\| v(\aleph_i) - v(\aleph_j) \right\|_a^2 \tag{7}$$

Where$\aleph_i$ and $\aleph_j$ define the square neighborhood window indices at pixels i and j respectively. So

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$v(\aleph_i)$ and $v(\aleph_j)$ define windows centered at pixels i and j respectively, with user-defined radius, r, and $W(i)$ defines normalization factor calculated as

$$W(i) = \sum_{j\epsilon S} \exp\left({}^{-d(i,j)}/_{h^2}\right) \quad (8)$$

Here S defines search width. Actual definition of Non Local Means conceives that each pixel can be associated or linked to all other pixels in an image, however practically numbers of pixels for weighted averaging are restricted to neighborhood called " search width ", s_i having size of $(2s+1)^2$ centered at pixel i.

The filtering parameter, h, controls exponential expression decay in weighting scheme. By choosing very small size filtering parameter, h, retains the noise while very large size smoothed the image. This means filtering parameter controls smoothing degree of filtered image.

Figure 3 shows the proposed higher order Non Local Mean filter suggested in this work

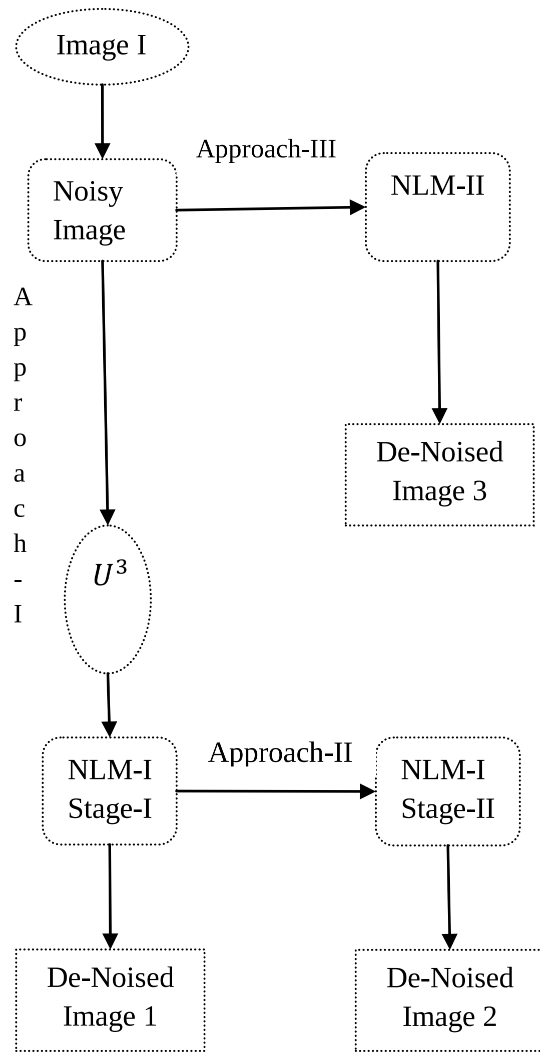

Figure 3: Proposed modified higher order Non Local Mean filter

In this work, NLM is applied with different variants for de-noising the image contaminated by Poisson noise. In the First approach (NLM-I Stage I) cube of image is taken and then NLM is applied to it. Mathematically the approach can be represented as in (9)

$$NL(U^3(i)) = \sum_{j\epsilon\aleph_{i^3}} w^3(i,j)U^3(j) \quad (9)$$

Whereas in second approach (NLM-I Stage II), which depends on NLM-I (Stage-I), square root of difference of NLM-I Stage I and square of

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estimated noise ($\tilde{\sigma}^2$). Mathematically it can be represented by equation (10)

$$Denoise\ Image = \sqrt{NL(U^3(i)) - \tilde{\sigma}^2} \quad (10)$$

However, in third approach (NLM-II) square of estimated noise ($\tilde{\sigma}^2$) is subtracted from cube of image and the square root is taken. Afterwards NLM is applied on it. Let I be Original image, I^3define cube of image and Ddefine square root of difference of cubic image I^3 and square of estimated noise, $\tilde{\sigma}^2$. Mathematically it may be represented by equation (11)

$$D = \sqrt{I^3 - \tilde{\sigma}^2} \quad (11)$$

Therefore, NLM of equation (11) is given as

$$NL(D) = NL(\sqrt{I^3 - \tilde{\sigma}^2}) \quad (12)$$

Which may also be represented as

$$NL\big(D(i)\big) = \sum_{j\epsilon\aleph_i} w(i,j)D(j) \quad (13)$$

4. Results and Discussion

In this work de-noising is performed by NLM embedded with some variations for applying on LENA, X-Rays, PET and SPECT images blurred by Poisson noise. The results of these variant approaches are compared with the results obtained from using Median and Wiener filters alone.

Figure 4 shows how noisy images of LENA, X-Rays, PET and SPECT can be de-noised transforming them close to original images by using NLM-I (Stage-I). Figure 5 shows images obtained by using NLM-I (Stage-II), NLM-II, Median and Wiener filters for de-nosing original images corrupted by noise. Figure 6 shows correlation results of noisy and recovered images for performance comparison of proposed and conventional filters for de-noising of images. De-noising results in the form of statistical parameters obtained with different de-noising techniques are tabulated in Table 1.

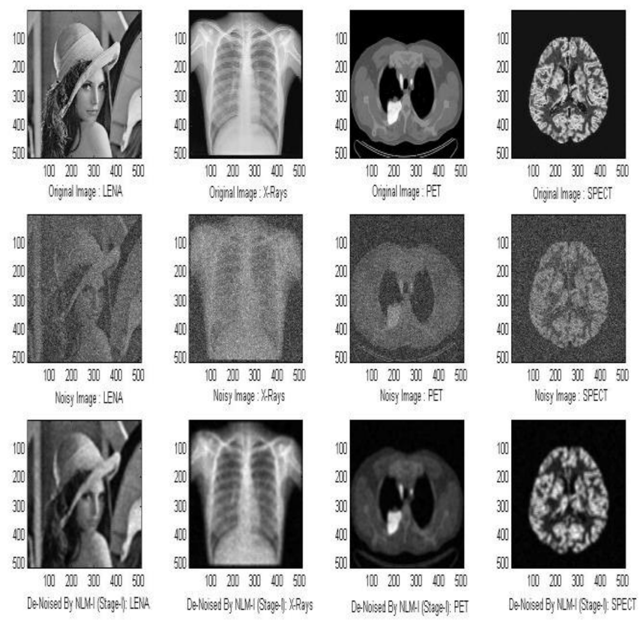

Figure 4: Original, Noisy and De-Noised Images by NLM-I (Stage-I)

The graphical and tabulated results of correlation for Lena show that the performance of NLM-II is excellent from NLM-I (Stage-I and Stage-II), Median and Wiener filters. However, the overall efficiency of NLM-I (Stage-I) and NLM-I (Stage-II) is same but seem better than Median and Wiener Filter.

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(Stage-II) is same but seem better than Median and Wiener Filter.

Visually it seems that Lena image de-noised by Median and Wiener filter is blurry and also Lena image de-noised by Wiener filter have noise at edges. Visual effects of NLM-I (Stage-I and Stage-II) and NLM-II are much better from Median and Wiener filter.

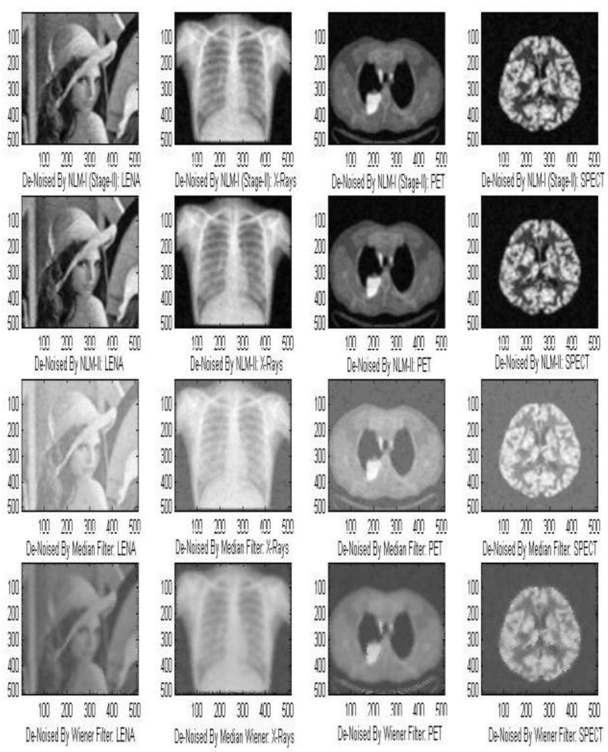

Figure 5: De-Noised Image by NLM-I (Stage-II), NLM-II, Median Filter and Wiener Filter

Moreover, the efficiency of NLM-I(Stage-II) and NLM-II for X-Rays image is more appreciable, as the images de-noised by NLM-I(Stage-II) and NLM-II have correlation better than what one obtain from NLM-I(Stage-I), Median and Wiener filter. However, efficiency of NLM-I(Stage-I) for X-Rays image is higher from those obtained with Median and Wiener filters. Furthermore, it is illustrated from correlation graph that efficiency of NLM-I(Stage-I and Stage-II) and NLM-II for X-Rays remains better for high noise element, whereas performance of Median and Wiener filter decreases with increasing values of noise. Visually X-Rays images de-noised by Median and Wiener filter are blurry and also Wiener filter leaves noise at edges. Visually NLM-I (Stage-I and Stage-II) and NLM-II have better results.

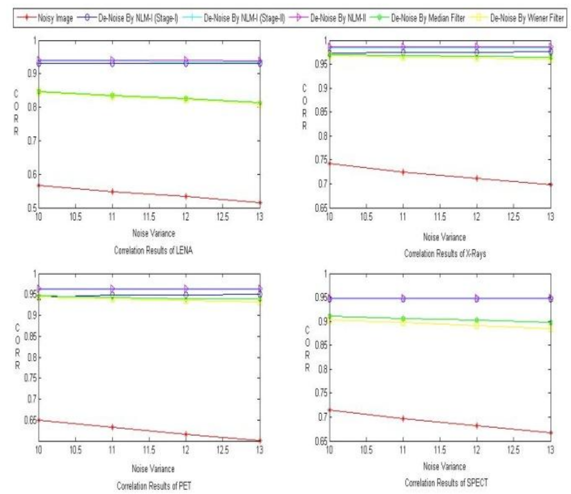

Figure 6: Correlation results of recovered images by proposed and conventional filters

Although the tabular and graphical results of correlation of PET image shows that efficiency of NLM-I (Stage-II) and NLM-II is more than NLM-I (Stage-I), Median and Wiener filters. Whereas Graphs shows that efficiency of NLM-I (Stage-I) is close to Median and Wiener filter. However its performance improves with increase of noise content. Visual results for PET image are same as X-Rays image.

While looking at the graphical and tabular results of Correlation for SPECT image it seems that the performances of NLM-I (Stage-I and Stage-II) and NLM-II are same and better than those by Median and Wiener filter. The performance of proposed variant structures increases with the increase in noise.

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Table 1: CORR for LENA, X-Rays, PET, SPECT

	Correlation					
	Noisy Image	NLM-I (Stage-I)	NLM-I (Stage-II)	NLM-II	Median Filter	Wiener Filter
LENA	0.5672	0.9305	0.9379	0.9407	0.8478	0.8444
X-Rays	0.7432	0.9731	0.9851	0.9863	0.9698	0.9675
PET	0.6501	0.9434	0.9621	0.9640	0.9462	0.9439
SPECT	0.7143	0.9468	0.9466	0.9488	0.9104	0.9029

5. Conclusion

Image restoration has done in order to improve the qualitative inspection of the image while estimating the performance parameters of the image analysis technique. Visual inspection is to determine the quality of the de-noised image, as the image should be clear and contain no artifacts or noise. However, in this work besides the visual, graphical and tabular results are achieved. It can be easily summarized that efficiency of NLM-I (Stage-I and Stage-II) and NLM-II is appreciably higher than those by Median and Wiener filter in all cases. However, NLM-II has better efficiency for LENA, X-Rays and SPECT images. For SPECT image NLM-I (Stage-I and Stage-II) and NLM-II have got approximately comparable efficiency. Thus it can be concluded that the higher order moment of the images enhance signal and noise, which can easily be de-noised and handled by NLM filter.

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