Analysis of the Rotating Magnetic Field of an AC Machines in the Transient Cases

Dr. Omar M. Albarbarawi.
Al-Balqa' Applied University, Faculty of Engineering Technology
Electrical Engineering Department, Amman- Jordan

Abstract

This paper deals with a study and analysis of the **R**otating **M**agnetic **F**ield (RMF) produced by currents flowing in the multiphase windings of an **I**nduction **M**otors (IM) in the transient cases. The Comparison is carried with normal operation when a multiphase set of currents of equal magnitude angular displacement between currents phases flows in multiphase winding with the same angular displacement between currents phases[1][2][3]. MATLAB software is utilized to study and analysis the behavior of the rotating magnetic field in the multi-phase stator in normal and abnormal AC motor **[6]**. This paper presents a step by step MATLAB software implementation to analyze the Rotating Magnetic Field which produces from multiphase currents for different cases.

Keywords- Symmetrical rotating magnetic field, asymmetrical rotating magnetic field, induction motor, split-phase, the current system, Mathematical models, MATLAB software, elliptical motion**.**

1. Introduction

The **R**otating **M**agnetic **F**ield (RMF) is considered as one of the greatest discoveries of Nikolay Tesla for all times, which forms the fundamental principle of operation of all AC machines, and also the basis of the operation of the Magnetic Resonance Imaging Technology (MRI) [2], [3]**,[8**]. The RMF arises when two-phase, or three-phase, or multiphase set of AC currents flowing in the two-phase, or three-phase, or multiphase windings installed around the inner surface of the machine. i.e RMF arises in the stator of AC machines in steady state condition, (when the number of windings phases in the stator are equal to the number of the applied phase currents, with the same angular displacement between the phases of the currents and the windings phases, which are equal to (360/ m.) electrical degrees (where N_{ph} = 3, 4, 5, 6,⋯m -number of phases).) Therefore if a balanced multi-phase set of AC currents, each of equal magnitude and displaced from each other by (360/m) electrical degree, flowing in the multi-phase windings with the same angular displacement as currents, it will produce an RMF with a constant magnitude. The field consists of two waves of a Magnetic Flux Density (MFD)[3] [4] [5] [7], distributed in shape of sinewave and cosine wave forms with maintaining a constant total magnitude at any time. Where the MFD resulted from the Magnetic Field Intensity (MFI) and the MFI resulted from the currents. Where the MFD rotates in a circular form in a clockwise or in a counterclockwise direction, and it will continue to rotate at an angular velocity (ωt) [5] [6][8].

While Asymmetrical Rotating Magnetic Field (ARMF) produces a transient cases which occurs when one phase or more phases of the currents fails, or when the angular displacement between the phases of currents or the windings phases are not equals, or when a short circuit between windings phases occurs. Thus when the currents at any one of these cases flows through a phases windings, it will produce an RMF distributed in a form of sine and cosine waves , with a variable total magnitude, and it will continue to rotate at an angular velocity (ωt) in an elliptical form.

When the wave of the phase current in a positive direction, the current is flowing from the beginning of the phase winding to the end, but when the wave of the phase current in a negative direction, the current is flowing from the end of the phase winding to the beginning. While the current doesn't flow in the phase winding when the instantaneous current equal zero.In this study, we will consider a two- pole machine with 3-phase windings on the stator.

2-Analysis of symmetrical rotating magnetic field

If three phase currents are equal in magnitude and the phase angle between them is 120 electrical degrees, they will produce a constant magnitude of rotating magnetic field. Assume that the currents flowing into 3-phase windings (fig.2a) are given from the following equations [14] **[15] [16] [17]**

$$\left.\begin{aligned} i_{AA'} &= I_m \sin(\omega t) \\ i_{BB'} &= I_m \sin(\omega t - 120^0) \\ i_{CC'} &= I_m \sin(\omega t - 240^0) \end{aligned}\right\} ,A \quad (1)$$

Where $i_{AA'}$, $i_{BB'}$, $i_{CC'}$ are the instantaneous stator currents. These currents produce a magnetic field intensities (MFIs) (fig.2b and 2c) are given by Equation (2):

$$\left.\begin{aligned} H_{AA'} &= H_m \sin(wt) \llcorner 0^0 \\ H_{BB'} &= H_m \sin(wt - 120^0) \llcorner 120^0 \\ H_{CC'} &= H_m \sin(wt - 240^0) \llcorner 240^0 \end{aligned}\right\} ,\ \text{A.turns/m}$$
(2)

Where $H_{AA'}$, $H_{BB'}$, $H_{CC'}$ are the instantaneous values of the MFIs for different phases.

The magnetic flux densities resulting from these magnetic field intensities are given by Equation (2):

$$\left.\begin{aligned} B_{AA'} &= B_m \sin(\omega t) \llcorner 0^0 \\ B_{BB'} &= B_m \sin(\omega t - 120^0) \angle 120^0 \\ B_{CC'} &= B_m \sin(\omega t - 240^0) \angle 240^0 \end{aligned}\right\} ,\quad \text{Wb/m}^2$$
(2)

where $B_m = \mu H_m$, (4) Where $B_{AA'}$, $B_{BB'}$, $B_{CC'}$ are the instantaneous value of the magnetic field intensities for different phases μ : -permeability of material

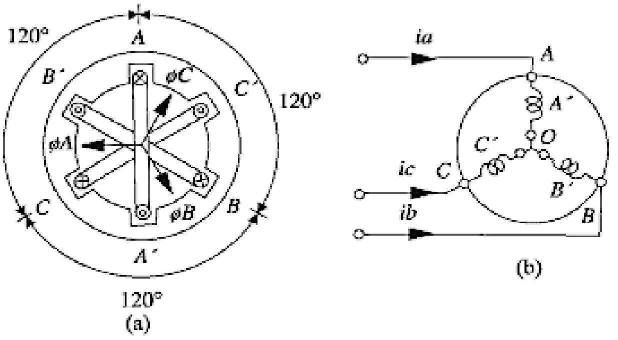


Figure 1. 3-phase current is flowing into a 3-phase winding

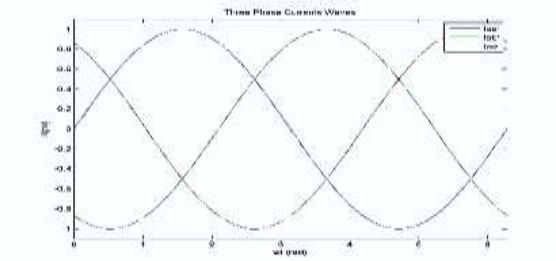
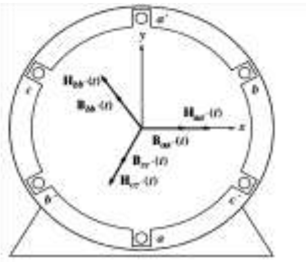


Figure 2. A balanced three-phase currents



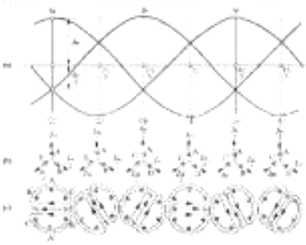


Figure 3. magnetic field waves of magnetic field (MFD vector B(t)) produced by each coil

Figure 4. The intensity vectors H(t) densities. (magnetic field intensity)

At any instant of time (t), the magnetic field will have the same magnitude which equals of 1.5 Bm, and continues to rotate at angular velocity ωt, as shown in figure (3-a).

The total magnetic flux density in the stator is the added vectors of all three components of the magnetic fields and determine their sum as shown below:

$$B_{net}(t) = B_{AA'}(t) + B_{BB'}(t) + B_{CC'}(t)$$

$$B_{net}(t) = B_m \sin(\omega t)\angle 0° + B_m \sin(\omega t - 120°)\angle 120° + B_m \sin(wt - 240°)\angle 240°,\ \ {}^{wb}/_{m^2}$$
(5)

As shown in figure (2b and 2c). Each of the three magnetic field components can be decomposed into its X & У components. By using the angle addition trigonometric identities, we can find the total magnetic flux density presented in the stator as follows

$$B_{net}(t)=0.5\,N_{ph}B_m\,[\sin(\omega t)\dot{x}-j\cos(wt)\hat{y}]$$

$$B_{net}(t)=0.5N_{ph}B_m \sin\angle(\omega t-\tfrac{\pi}{2}) \quad {}^{wb}/_{m^2} \qquad (6)$$

where N_{ph} =2, 3, 4, 5 …, N_{ph} - number of phases.

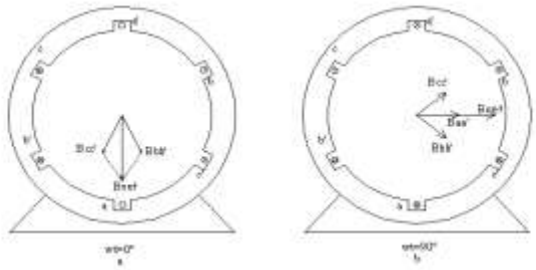


Figure 5. The magnetic field density vector in a stator at time ωt = 0°, and at time ωt = 90°

This equation is the final expression for the total magnetic flux density produced in the stator. It consists of two parts changing in sinusoidal and cosine waves as shown in figure (3b). Notice that the field is a constant value equal ($0.5N_{ph}B_m$) and rotates in a circular motion with continuously changed angle at angular velocity ωt, and begins to rotate at angle $\angle$-90^0 as shown in figure (7).

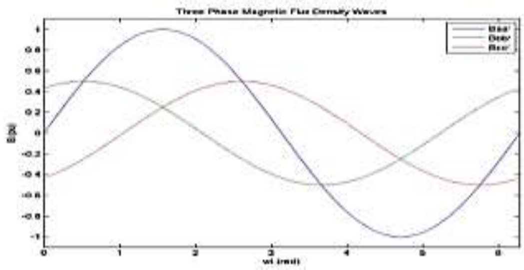


Figure 6. The sine, cosine and the net sinusoidal waves of magnetic field density

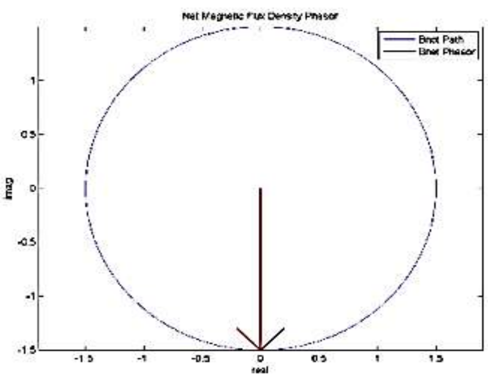


Figure 7. The natural rotating magnetic field in circular form for a balanced of 3-phase currents.

3. Reversing the direction of magnetic field rotation

Another reality that can be observed about the resulting magnetic field, If we interchanging the connection of any two of a three phase stator currents , the direction of the magnetic field's rotation will be reversed (clockwise direction) with the same magnitude and the same angular velocity (ωt), as above, and begins to rotate from angle $\angle 90^O$ in a circular motion, as shown in Figure. 8

This means that it is possible to reverse the direction of rotation of an AC motor just by interchange the connection of any two of the three coils of the power supply.

To proof the reversing direction speed of the motor or the rotation field that come by interchanging the connections of the phases bb' and cc' we will use the following equations for the net magnetic flux density in the stator:-

$$B_{net}(t)=B_{AA'}(t)+B_{BB'}(t)+B_{CC'}(t)$$

$$B_{net}(t)=B_m\sin(\omega t)\angle 0°+B_m\sin(\omega t\text{-}240°)\angle 120° + B_m\sin(\omega t\text{-}120°)\angle 240°, \qquad (6)$$

These components of the magnetic fields can be broken down into its x and y- component's, then combining x and y components with each, to determine the resultant total of the field as the following:-

$$B_{net}(t)=0.5\,N_{ph}B_m\,[\sin(wt)\dot{x}+j\cos(wt)\hat{y}] \quad .$$

$$B_{net}(t)=0.5N_{ph}B_m\sin\angle(+\tfrac{\pi}{2}-wt) \quad {}^{wb}/_{m^2} \qquad (7)$$

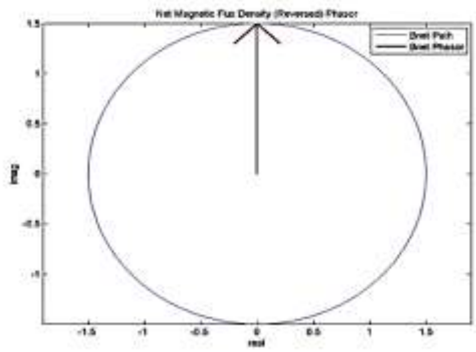


Figure 8. Reversing (clockwise) direction of the RMF

This means that it is possible to reverse the direction of the motor rotation. in a circular motion and begins to rotate from angle $\angle 90^o$ as shown in Figure 8.

Improvement the electromagnetic torque and the starting torque

The performance of the induction motor with a split - phase can be improved due to the increasing of the electromagnetic power, the electromagnetic torque and the starting torque. Dividing each phase in the stator into two equal parts with 90^o angular displacement and supplying these split-phases with a 3-phase set currents by using Equations(1) , will result in a magnetic flux density with the following total

$$B_{net}(t)=B_{AA'}(t)+B_{BB'}(t)+B_{CC'}(t) \quad ,So$$

$$B_{net}(t) = B_m\,[\sin(\omega t)\angle 0° + \sin(\omega t)\angle 90°]$$
$$+ B_m\,[\sin(\omega t\text{-}120°)\angle 120° + \sin(\omega t\text{-}120°)\angle 210°] +$$
$$+ B_m\,[\sin(\omega t\text{-}240°)\angle 240° + \sin(\omega t\text{-}240°)\angle 330°]$$
$$B_{net}(t)=3B_m\cos(\tfrac{\alpha}{2})[\cos(\omega t\text{-}45°)+j\sin(\omega t\text{-}45°)]$$
.

$$B_{net}(t)=3Bm\cos(\tfrac{\alpha}{2})\angle(\omega t\text{-}45°)\ {}^{wb}/_{m^2}$$
(9)

Where α - angular displacement between the partial split – phase which equals (90^0).

Also we can use a three-phase induction motors with a split - phase by dividing each phase into two equal parts, with angular displacement between them equals to (60^0), and feeding these split- phases with a 3-phase set currents by using the equations (1) . From these currents, The following total magnetic flux density in the stator arises

$$B_{net}(t)=B_{AA'}(t)+B_{BB'}(t)+B_{CC'}(t), So$$
$$B_{net}(t)=B_m\,[\sin(\omega t)\angle 0° + \sin(\omega t)\angle 60°]+$$

$$+ B_m\,[\sin(\omega t\text{-}120°)\angle 120° + \sin(\omega t\text{-}120°)\angle 180°]+$$
$$+ B_m\,[\sin(\omega t\text{-}240°)\angle 240° + \sin(\omega t\text{-}240°)\angle 300°]+$$
$$B_{net}(t)=3B_m\cos(\tfrac{\alpha}{2})\ [\cos(\omega t\text{-}60°)+j\sin(\omega t\text{-}60°)]\ {}^{wb}/_{m^2}$$
$$B_{net}(t) = 3B_m\cos(\tfrac{\alpha}{2})\angle(\omega t\text{-}60°)\ \ {}^{wb}/_{m^2}$$
.(11)

Where α - angular displacement between the partial split - phase, and here this angle is equal to (60^o). The above equations (10 and 11) is the final expression for a total magnetic flux density presented in the stator, which produced on split - phases, and consists of two parts changing in cosine and sine waves with constant magnitude equals (2.2132 Bm) for the split - phase with a shift angle (90^o), and equals (2.6 Bm) for the split - phase with a shift angle (60^o), and rotates in a circular motion. They will continually change at angular velocity ωt with phase shift angle of $\angle$-60^0.

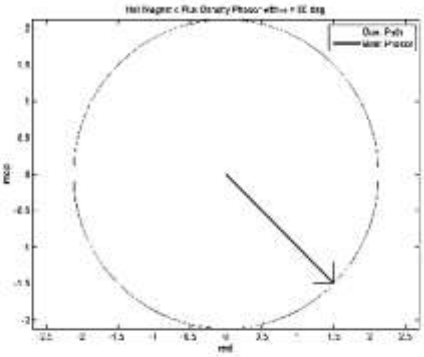


Figure 9. rotating magnetic field in case of a . split-phase with angle of 90^0

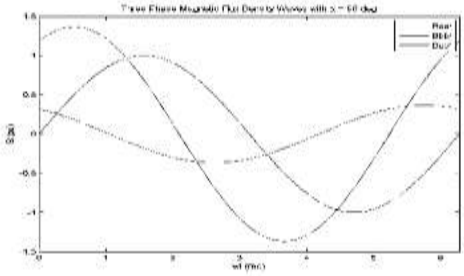


Figure 10. The sine and cosine waves and the net waves of magnetic . field density with split-phases.

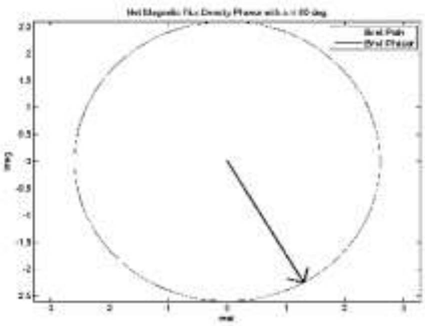


Figure11. Rotating magnetic field in case of a split-phase with angle of 60^0.

The rotating magnetic field magnitude in a split-phases case with an angle of 120^0 is not different from the field magnitude in a three phase without split-phases (normal) case, but it begins to rotate from the angle of (-30^0) . The value of this field can be determined by the following equation :

$$B_{net(t)}=1.5B_m \llcorner(-\pi/6+wt), wb/m^2$$

Analysis of asymmetrical rotating magnetic field

If one of the three phases in induction motor stator is disconnected, the other two phase currents will produce a magnetic field consisting of sine and cosine wave forms resulting in a rotating elliptical form with variable magnitudes.

Suppose if a phase of a ($i_{AA'}$)current is disconnected and the other two phases of currents ($i_{BB'}$,$i_{CC'}$) are remained, then the equation set can be written as :

$$\left.\begin{array}{l} i_{BB'} = I_m \sin(wt - 120^0) \\ i_{CC'} = I_m \sin(wt - 240^0) \end{array}\right\} A$$

They will produce the following magnetic flux density

$$\left.\begin{array}{l} B_{BB'}(t)=B_m \sin(wt\text{-}120°)\angle 120° \\ B_{CC'}(t)=B_m \sin(wt\text{-}240°)\angle 240° \end{array}\right\} ,W/m2 \quad (6)$$

The total magnetic flux density presented in the stator consists of two parts changing in sine and cosine waves and rotates with an elliptical form can be written as

$$B_{net(t)} = B_{BB'(t)} + B_{CC'(t)}$$

$$B_{net}(t) = 0.5\ B_m \sin(wt) + 1.5\ B_m \cos(wt)\angle\text{-}\pi/2\ {}^{wb}/_{m^2} \quad (15)$$

The field begins to rotate in an elliptical form, from angle of ∠-90^0 where the field lines are concentrated along the axis of the phase AA′ as shown in figure 12.
As mentioned above, if the phase of current ($i_{BB'}$) is disconnected, and others two phases remained to feed the motor stator, they will produce a total magnetic flux density presented in the stator as follows

$$B_{net(t)} = B_{AA\prime(t)} + B_{CC'(t)}$$

$$B_{net}(t)=B_m \sin(wt)+ B_m \cos(wt - 150^0)\angle 60^0, wb/m^2$$

The field begins to rotates with an elliptical from at angle of ∟-120^O, and its field lines are concentrated along the axis of the phase CC′ . as shown in figure 13
As mentioned above, if the phase of current ($i_{CC'}$) is disconnected , and others two phases remained to feed the motor stator, they will produces a total magnetic flux density presented in the stator as follows

$$B_{net}(t)=B_{AA'}(t)+B_{BB'}(t)$$

$$B_{net}(t)=B_m \sin(wt)+ B_m \cos(wt\text{-}30°)\angle\text{-}60°\ {}^{wb}/_{m^2}$$
(21)

The field begins to rotates with an elliptical from at angle of ∟-60^O, and its field lines are concentrated along the axis of the phase BB′ .
From the above analysis, we can note that if any one phase of the three phase motor is disconnected, the total magnetic flux density presented in the stator and produced from the other two phases currents, will consist of two parts changing in sine and cosine waves, and rotates clockwise or counterclockwise with an elliptical form . The total value of the density will vary from 0.5 Bm at middle zone of the ellipse up to 1.5 Bm at vertices zone of the same ellipse while the field lines are concentrated on vertices zone of the ellipse and weakened on middle zone of the same ellipse as shown in figures(12, 13) For this reason the motor speed will decrease and become irregular.
From this condition, we can use this case as a stopping method of the motors, by separating any one phase current from a 3-phase currents which feed the motor and then connect this phase with electromagnetic brake coils, which installed on the face of the vertices zone of the ellipse, which has a highest magnetic attraction force in the motor

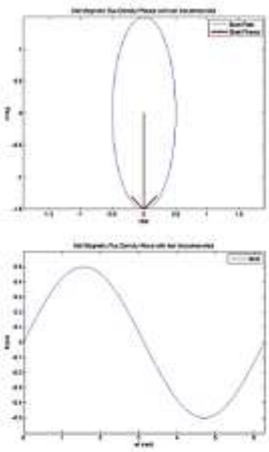


Figure 12. The resultant wave and space vector form of the rotating field when fail phase current $i_{AA'}$.

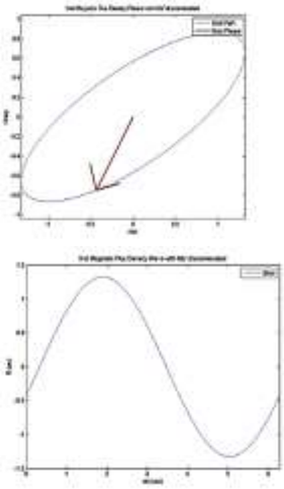


Figure 13. The resultant wave and space vector form the of rotating field when fail phase current $i_{BB'}$.

6. Two phase currents are fail

If a two phase currents are disconnected, and the other phase remained to feed the motor stator. They will produce a total magnetic flux density presented in the stator as follows

$B_{net}(t)=B_m \sin(wt)\,,\ {}^{wb}/_{m^2}$ (21)

The field begins its alternative motion in a linear fashion as shown in figure 14.

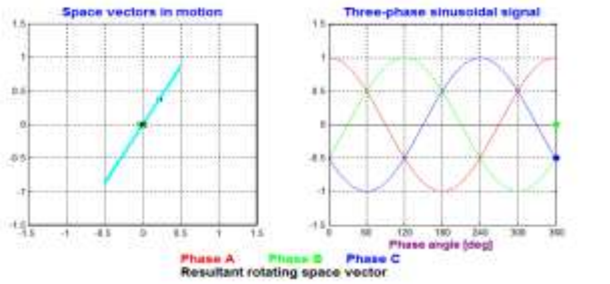


Figure 14. Resultant space vector when fail two phase currents.

From the above if any phase current separated from the power supply, this will decrease the magnetic flux for more than 33% at its rated value, and also lead to decrease the generated EMF in the stator which will increase the stator currents.

Rotating magnetic field in a single-phase induction motor.

The single phase induction motor with a split-phase can be used with two stator windings[2]; main winding and start winding, where the phase shift between them is equal to 90^0 and the supply source is a single phase or two phase currents equal in magnitude and the differ in phase. (the phase between them is 90° and can be satisfied by using thyristors). The currents equations are

$$i_{AA'}=I_m \sin(it)\,, A$$
$$i_{BB'}=I_m \sin(wt\text{-}90°)\,, A \qquad (22)$$

These currents will produce a total magnetic flux densities as the following

$$B_{net}(t)=B_{AA'}(t)+B_{BB'}(t)$$
$$Bnet(t)=B_m \sin(wt)\angle 0°+ B_m \sin(wt\text{-}90°)\angle 90°$$
$$Bnet(t) =B_m\angle(wt\text{-}90°),\ {}^{wb}/_{m^2}$$

(24)

At any instant of time (t), the magnetic field will have the same magnitude equals of Bm rotates in a circular motion from the angle of $\angle$-90^o and continues to rotate at angular velocity ωt, as shown in figure (3-a).

Split-Phase with Angular Displacement Between Windings 120^0 and Between Currents is 90^0

To increase the starting torque in a single phase induction motor, the single-phase induction motor with a split-phase can be used with two stator windings[2]; main winding and start winding, where the phase shift between them is equal to 120^0 and the supply source is a single phase or two phase currents equal in magnitude and the differ in phase. (the phase between them is 90° and can be satisfied by using thyristors). The currents equations are

$$i_{AA'}=I_m \sin(wt)\,, A$$
$$i_{BB'}=I_m \sin(wt\text{-}90°)\,, A$$

(25)

These currents will produce a total magnetic flux densities as the following:

$$B_{AA'}(t)=B_m \sin(wt)\angle 0°,\ {}^{wb}/_{m^2},$$
$$B_{BB'}(t)=B_m \sin(wt\text{-}90°)\angle 120°,\ {}^{wb}/_{m^2}$$
$$B_{net}(t)=B_m \sin(wt)\angle 0°+ B_m \sin(wt\text{-}90°)\angle 120°$$
$$Bnet(t) = B_m[sin(wt) + cos(wt)\angle - 60°)],{}^{wb}/_{m^2}$$

The total magnetic field begins to rotate in an elliptical form at angle of ∟-60^O, and its field lines are concentrated along the axis of the starting winding, or if the angular displacement between the main winding and the starting winding is 90^0, and they are feeding from a supply of two currents equal in magnitude and differ in phase by 120^0 (the phase between them is 90° and can be satisfied by using thyristors) , The currents equations are

$$i_{AA'}=I_m \sin(wt)\,, A$$
$$i_{BB'}=I_m \sin(wt\text{-}120°)\,, A \qquad (27)$$

From these currents will produce total magnetic flux densities as the following

$$B_{AA'}(t)=B_m \sin(wt)\angle 0°,\ {}^{wb}/_{m^2},$$
$$B_{BB'}(t)=B_m \sin(wt\text{-}120°)\angle 90°,\ {}^{wb}/_{m^2}$$
$$B_{net}(t) = B_{AA'}(t) + B_{BB'}(t)$$

(28)

$$B_{net}(t) = B_m \sin(wt)\angle 0 + B_m \sin(wt - 120°)\angle 90°$$
$$B_{net}(t) = Bm\,[\sin(wt) + \cos(wt - 30°)\,\angle 90°)]\ wb/m^2$$

The total magnetic field begins to rotate an elliptical form, at an angle of $\angle -90°$, where the field lines are concentrated along the axis of the starting winding.

From the above two cases of the single phase with a split-phase, the total magnetic flux density present in the stator consists of two parts, which are changing in a sine and cosine waves, rotate in an elliptic form, with a total variable value from 0.7071 Bm in the middle zone of the elliptic up to 1.225 Bm in vertices zone of the ellipse, where the field lines are concentrated in the vertices zone and weaken in the middle zone of the ellipse.

Conclusion

This paper is an attempt to conclude that the study and analysis of the rotating magnetic field are very important to analyze the performance and the operation of induction motors in both normal and abnormal conditions. If multiphase currents are equal in magnitude and have the same angular displacement between them, they will produce a constant rotating magnetic field magnitude. Assume that these currents flowing into a multiphase winding with the same angular displacement as the phase currents, it will produce a rotating magnetic field that consists of a sine and cosine waves which rotate inside of the machine in a circular motion with constant magnitude equal to $0.5N_{ph}Bm$. The sine and cosine waves will form a net magnetic flux density distributed in sinusoidal waves which generate the EMF distributed in a sine wave in the stator and rotor that is similar to the transformers (see figures 6] - [7]).

Improving the function of the AC machine and increasing its power, torque and starting torque can be done through using multiphase phase windings with split-phases with an angular displacement between the split-phases equals to 60 or 90 electrical degrees. Thus when a balanced three-phase set currents, flowing through the 3-phase stator windings with split-phase will increase the total produced magnetic flux density in the stator , that consist of sine and cosine waves, with constant magnitude of more than 1.5Bm , and rotate in a circular motion , with continuously changed angular velocity (ωt) at any time (t).. see figures [9] - [10]

If the motor worked in an abnormal conditions such as different angular displacement between phase currents and windings phases, one current phase separated from the power supplying and the others two phases are remained to feed the motor, or if the supplying voltages unbalanced; it would arise a magnetic field in the stator that consist of two parts, distributed in a sine and cosine waves, which formed RMF , and rotates in an elliptic form while the field lines are concentrated in a vertices zone of the ellipse and weakened in middle zone of the same ellipse, For this reason, the motor speed will decrease and become irregular,. For this condition, we can use this case as a stopping method of the motors, by connecting the separated phase with electromagnetic brake coils and at the same time using the plugging braking by interchanging connections of the others two phase currents.

To increase the starting torque of the single phase induction motor with two winding (main and start) with an angular displacement between them 90^0 , we can feed its windings by currents with 120^0 displacement angle between the feeding currents, also we can increase the starting torque by using angular displacement between winding of 120^0 and feeding this winding by currents with 90^0 displacement angle between the feeding currents.

References

[1] The Rotating Magnetic Field Theory of A-C. Motors. IEEE Xplore. (Volume: XLIV) June 2009 , Pages 340 -348.

[2] The rotating magnetic field theory of A-C motors Browse Journals &Magazines Journal of the A.I.E.E. Volume: 44 Issue: 2

[3] Production of rotating magnetic field, electrical easy.com

[4] Production of rotating magnetic fields in polyphase AC machines: a novel teaching approach S Panthera - AU J Technol , 105 -110 (Jan. 2004) - journal.au.edu

[5] W. Mahoney, Jake J. Abbott, Generating Rotating Magnetic Fields With a Single Permanent Magnet for for Propulsion of Untethered Magnetic Devices in a Lumen IEEE, VOL. 30, NO. 2, APRIL 2014 411,

[6] J.Holtz; T. Thimm production of rotating magnetic. Field in polyphase stator.

[7] HTTP:// www.scribd.com 8.3 Rotating Magnetic Field Due to 3-Phase Currents for Principles of Electrical Machines (V.K MEHTA)

[8] Tesla, Nikola; AIEE Trans. (1888). "A New System for Alternating Current Motors and. Transformers". AIEE. 5: 308–324. Retrieved 17 December 2012.

[9] Jump up Thompson, Silvanus Phillips Polyphaser Electric Currents and Alternate-Current Motors (1st ed.). London: E. & F.N. Spon. p. 261. Retrieved 2 December 2012.

[10] Induction motor , From Wikipedia , the free encyclopedia

[11] Encyclopedia Americana: Meyer to Nauvoo, Scholastic Library Pub., 2006, page 558 . On line sites resources
[12] http://www.en.wikipedia.org/wiki/Rotating_ magnetic _field Rotating magnetic field - Wikipedia, the free encyclopedia
[13] Encyclopedia Americana: Meyer to Nauvoo, Scholastic Library Pub., 2006, page 558 . On line sites resources
[14] Rotating magnetic field - Wikipedia https://en.wikipedia.org/wiki/RMF.
Book Reference
[15] Stephen J, Chapman: " Electric Machinery Fundamental ," McGraw-Hill Inc.
[16] George McPherson: "An Introduction to Electrical Machines and Transformers", John . Wiley & sons.
[17] P. S. Bimbhra: "Electrical Machinery" , Khanna. Publishers.
[18] Paresh C. Sen: " Principles of Electric Machines and. Power Electronics" , John Wiley & Sons.
[19] Electrical machines A. Ivanov-Smolinsky Mir publishers Moscow.
[20] D. P. Kothari & I.J. Nagrath , Tata McGraw Hill & 1995," Electrical Machines".