Nonlinear dynamic response of flexible mechatronic system with clearance

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Abstract: *This work is an investigation of the effects of link flexibility and clearances on the dynamic behavior's mechatronic system. A flexible four bar mechanism with joint clearance between crank and coupler connected to a controlled DC motor is studied. The dynamic equations of motion are derived by application of Lagrangian formulation and then they are modeled by using Simulink®. The simulations show the effect of flexible mechanical subsystem with clearance on the performance of a mechatronic system.*
Keywords: *Mechatronic system, Flexible four bar mechanism, Joint clearance, Matlab/Simulink®, Lagrange's equation, Euler-Bernoulli beam.*

1. Introduction

The clearances always exist in the joints of mechanism owing to tolerances and defects arising from design and manufacturing process or wearing after a certain working period. However, the presence of excessive joint clearances degrades the performances of the mechanism and generally leads to the presence of raised constraints, vibration and noise. Moreover, the high productivity and precision demanded by modern industry require very high-speeds, light-weights and high-precision machinery. So the study of dynamic response of the elastic linkage mechanisms becomes more and more important.

Many researchers have studied the performance of the elastic linkage mechanisms with clearance. However, they have studied dynamic response of these mechanisms without considering the corresponding control system.

Dubowsky et al [1] investigated the effects of clearances and link flexibility on the stresses in joints of high speed mechanisms. They developed a dynamic model using perturbation co-ordinate to predict the behavior of a single link subjected to dynamic excitation normal to its longitudinal axis. Then they developed in [2], the model of the general planar mechanisms with elastic links and multiple clearance connections. They applied their analytical approach to a scotch yoke and a four bar mechanism. Khemili and Rhomdane [3] were interested in the study of the dynamic behavior of a planar flexible slider crank mechanism with clearance. They built the model under the software ADAMS®. The presented results showed that, in the presence of clearance, the mechanical system response was greatly influenced and that the coupler flexibility had a role suspension for the mechanism. Salahshoor et al [4] investigated the nonlinear vibrational analysis of a sliding pendulum with single and multiple joint clearances. They used the method of multiples scales to analytically solve the problem. Yan and Guo [5] presented a general method of kinematic accuracy analysis for the flexible planar mechanisms with stochastic parameters such as with stochastic parameters such as uncertain link lengths and joint clearances. Erkaya and Usmay [6] investigated the dynamic characteristics of a planar four-bar mechanism having joint clearance and link

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flexibility. They analyzed the effects of clearance on planar mechanism having rigid and flexible links.

Others researchers have investigated the dynamic behavior of the electrical subsystem. For instance, Omid et al [7] examined the performance of two drives for an electric vehicle application. To simulate an electric or hybrid-electric vehicle, a stepping torque profile was simulated with both drives in Simulink. Affi et al [8] presented a genetic algorithm based method to design a mechatronic system. They optimized the geometry and dynamic behavior of an ideal four bar mechatronic system. A global optimization was presented as a multi-objective optimization one where the geometry and the dynamics were considered simultaneously. Ricardo et al [9] studied a four-bar linkage having a variable input velocity. The velocity of the crank was controlled in order to obtain the desired output motion at the coupler point.

In recent years, the researches have been carried out in order to investigate the dynamic behavior of flexible mechanism with clearance taking the mechanical and the electrical subsystems as an integrated system.

Bauchau and Rodriguez [10] were interested in modeling of joints with clearance in flexible multibody systems. They developed a formulation within the frame work of energy preserving and decaying time integration schemes for nonlinear flexible multibody systems. They studied the behavior of the crank slider mechanism (with a planar revolute joint and with a spatial clearance element) driven by a DC motor assuming the crank speed constant in some cases and variable in the other cases studied.

Chunmei et al [11] investigated the effect of clearance and elasticity on the dynamic behavior of a flexible four bar mechanism. The dynamic equations of motion were derived using Newton’s second law by choosing two sets of generalized co-ordinates. The dynamic equation of the flexible elements deformation was obtained using the finite element method. They analyzed the dynamic response of the mechanical system through an example assuming the angular velocity of crank to be variable in the operation. Both the effects of elasticity and clearance on the dynamic behaviors of the mechanism were analyzed simultaneously.

Cai et al [12] studied the nonlinear coupling dynamic model of a three phase AC motor linkage mechanism system with links fabricated from symmetric laminates. The dynamic equations of the motion were derived taking the drive motor and mechanism as an integrated system. A four-bar linkage mechanism with links fabricated from symmetric laminate was presented as an example. The transversal elastic displacement of the coupler midpoint was analyzed for the mechanism without consideration of the effect of the electromotor and the same mechanism coupled to the three phase AC motor. The simulation results showed that the electromotor had more significant effects on the dynamic responses of the system.

Erkaya and Usmay [13] proposed an experimental study to investigate the effect of balancing and link flexibility on the dynamics of a mechanism with imperfect revolute joint. An experimental test rig was set up, and bearing vibration was measured to determine the positive reflections of link flexibility and balancing on mechanisms having joint clearances.

Zhao et al [14] investigated the effects of flexibility at different locations on a mechanism on the wear at a clearance joint as well as the dynamic performance of a multibody mechanical system. A numerical approach was proposed for the modeling and prediction of wear at a revolute clearance joint in a flexible multibody mechanical system by integrating the procedures of wear prediction with multibody dynamics. Using this approach, a planar slider–crank mechanism including a clearance joint was used as an illustrative case. The effects of the flexibilities of a connecting rod and crankshaft on the clearance joint wear were compared.

From the above survey it reveals that, the researchers were limited to study the dynamic behavior of the mechanical subsystem having joint clearances and elastic links.

In this paper, we propose to study the effect of flexibility and clearance on the performance of the electrical subsystem of a mechatronic system. A planar flexible four-bar mechanism with joint clearance connected to a controlled DC motor is considered as an example. In order to take into account the parameters of the electrical subsystem, the current is analyzed rather than the torque. The current required for the mechanism to maintain a constant crank speed are defined analytically and then are implemented by using Simulink. The simulation results show the flexibility and clearance influences on the dynamic performance of the system. For this purpose, this paper is organized as follows: section 2 describes the mechanical system and the joint clearance model. In section 3 the driving motor model and the coupling equations between the electrical and mechanical subsystems are presented. The simulations results are given in section 4. Finally conclusions are outlined in section 5.

2. Dynamic equations of motion

This section presents the dynamic equations of the flexible four bar mechanism with clearance at joint between crank and coupler links (Fig. 1). The flexibility is included in the dynamic model of each link by use of an Euler-Bernoulli beam idealization.

2.1. Joint clearance

In the planar revolute joint, as shown in Fig.2, the joint clearance is idealized as the difference of the diameters of the pin and hole of a joint. Assuming constant contact between the pin and hole of a joint, a joint clearance may be modeled as a small link (called clearance link) with the length equal to one half of the joint clearance [15-16]. If the friction is negligible, the direction of the clearance vector coincides with the normal direction of the collision plane.

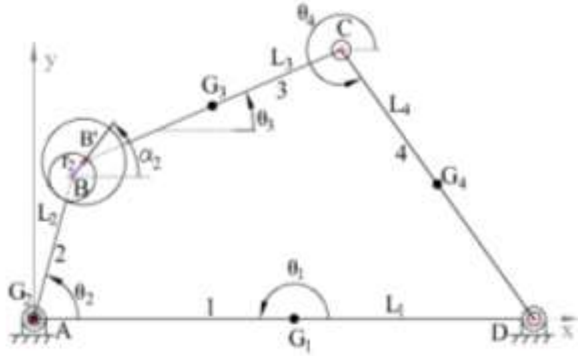

Fig. 1. A four-bar linkage with clearance

Each joint clearance adds one DOF to the linkage. The direction uncertainty due to the joint clearance is thus represented by the added redundant freedom in the model.

NOMENCLATURE

ω_2	Angular velocity of input link	I_i	Inertial moment of related link
α_2	Angular direction of joint clearance	i	Current
x_G, y_G	x- and y- coordinate of the center of mass	G_i	Center of mass of related link
U	Potential energy	g	Acceleration of gravity
T_m	Magnetic motor torque	e	electromotive force
T_l	Constant mechanical load	DF	Dissipation function
T	Torque	$C_{\theta i}$	Damping coefficient of related joint
r_2	Joint clearance link	$C_{\alpha 2}$	Damping coefficient of joint clearance
R	Armature resistance	B	Damping coefficient
m_i	Link mass of related link	θ_i	Angular direction of related link
L_i	Link length of related link	$\dot{\theta}_i$	Angular velocity of related link
L	Inductance	$\ddot{\theta}_i$	Angular acceleration of related link
K_m	Motor torque constant	J	Combined inertial moment of the motor and the gear system
K_g	Motor voltage constant		
K	Kinetic energy		

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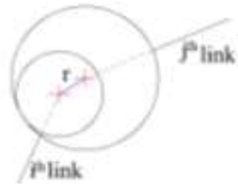

Fig. 2. A clearance link

2.2 Dynamic equations of the mechanism

In this application, it is assumed that the link deformation is sufficiently limited to allow the use of small deflection approximation as embodied in Euler beam theory, and that the link is rigid in the axial direction and that finally, there are no external forces or torques on the link except those acting at the end connection.

The deformation of the j link wj (Fig.3), with respect to the actual link center line, is expanded in a series of S independent functions. Selection of this bending function is somewhat arbitrary, experience having shown that a large class of function will give satisfactory results. Function that has no deflection at the ends will not be acted upon by the bearing forces an attractive feature since the total beam deflection need not be evaluated at each step in the calculation. The Fourier sine series are the most obvious choice, and, if the link is uniform, it simplifies the resulting equations. If the link is not uniform, the sine series may still be used. If they are known the pinned-pinned normal modes of the link, the same simplification will result [17], [18].

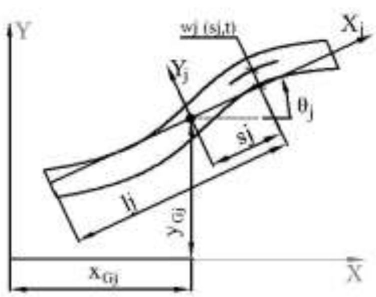

Fig. 3. Deformation representation of the j link

The expansion is written as

$$\vec{w}_j(s_j,t) = w_j(s_j,t)\vec{Y}_j = \sum_{i=1}^{s} \phi_j^i(s_j) q_j^i(t) \vec{Y}_j \qquad (1)$$

Where ϕ_j^i is the i[th] assumed displacement function for the j[th] link and q_j^i is its associated generalized coordinate.

The dynamic equations of the system are written by using Lagrange's equation.

$$\frac{d}{dt}\left[\frac{\partial K}{\partial q_i}\right] - \frac{\partial K}{\partial q_i} + \frac{\partial V}{\partial q_i} = F_i \qquad (2)$$

Where K and V are the kinetic and potential energy of the system and q_i and F_i are the i[th] generalized coordinates and forces.

The generalized coordinates in this application consist of the crank angle, the clearance direction and the elastic deformation variables

The kinetic energy K in equation (2), is equal to the sum of the kinetic energies of each of the four links in the system or

$$K = \frac{1}{2}\sum_{j=1}^{4}\int_0^{L_j} \vec{\dot{R}}_j . \vec{\dot{R}}_j \sigma_j dl_j \qquad (3)$$

Where R_j is the position vector of a typical masse element, dm_j, on the j[th] link in inertial space, and $\dot{R}_j$ is the inertial velocity of such a point.

$$\begin{aligned}\vec{R}_j &= (x_{Gj}\cos\theta_j + y_{Gj}\sin\theta_j + s_j)\vec{X}_j + \\ &\quad (-x_{Gj}\sin\theta_j + y_{Gj}\cos\theta_j + w_j)\vec{Y}_j\end{aligned} \qquad (4)$$

And

$$\begin{aligned}\vec{\dot{R}}_j &= (\dot{x}_{Gj}\cos\theta_j + \dot{y}_{Gj}\sin\theta_j - w_j\dot{\theta}_j)\vec{X}_j + \\ &\quad (-\dot{x}_{Gj}\sin\theta_j + \dot{y}_{Gj}\cos\theta_j + s_j\dot{\theta}_j + \dot{w}_j)\vec{Y}_j\end{aligned} \qquad (5)$$

Upon substitution of equation 5, eq. 3 becomes:

$$K = \frac{1}{2}\sum_{j=1}^{4}\int_0^{L_j} \sigma_j \left[\begin{aligned}&(\dot{x}_{Gj}\cos\theta_j + \dot{y}_{Gj}\sin\theta_j - w_j\dot{\theta}_j)^2 + \\ &(-\dot{x}_{Gj}\sin\theta_j + \dot{y}_{Gj}\cos\theta_j + \dot{w}_j + (l_j - \frac{L_j}{2})\dot{\theta}_j)^2\end{aligned}\right] dl_j \qquad (6)$$

The total strain energy is given by

$$V = V_{déf} + V_g$$

$$= \frac{1}{2}\sum_{j=1}^{4}\left(\begin{aligned}&\int_0^{L_j}\left[EI_j(l_j)\left(\frac{\partial^2 w_j}{\partial l_j^2}\right)^2 + P_j(l_j)\left(\frac{\partial w_j}{\partial l_j}\right)^2\right] dl_j \\ &+ (m_j Y_{Gj} + \cos\theta_j \int_0^{L_j} \sigma_j w_j dl_j)\end{aligned}\right) \qquad (7)$$

Where $V_{déf}$ is the energy of elastic deformation, V_g is the gravitational potential energy, l_j is the center line coordinate of the link measured from the left end, P_j is the axial tension and EI_j is the flexural rigidity, all of the j^{th} link. For the analysis of real machines the axial load term can be neglected [19, 20].

Substituting eq. 1, 6 and 7 into equation 2 may now be carried out and leads to the following set of ordinary, highly nonlinear, differential equations of motion.

$$A_1\ddot{\alpha}_2 + A_2\dot{\alpha}_2^2 + A_3\dot{\alpha}_2 + A_4 + \sum_{j=2}^{4}\sum_{i=1}^{s}(\ddot{q}_j^i B_j^{1i} + \dot{q}_j^i B_j^{2i} + q_j^i B_j^{3i}) = 0 \tag{8}$$

$$C_1\ddot{\alpha}_2 + C_2\dot{\alpha}_2^2 + C_3\dot{\alpha}_2 + C_4 + \sum_{j=2}^{4}\sum_{i=1}^{s}(\ddot{q}_j^i D_j^{1i} + \dot{q}_j^i D_j^{2i} + q_j^i D_j^{3i}) = T \tag{9}$$

$$E_j^{1i}\ddot{\alpha}_2 + E_j^{2i}\dot{\alpha}_2^2 + E_j^{3i}\dot{\alpha}_2 + E_j^{4i} + \ddot{q}_j^i E_j^{5i} + +2\dot{q}_j^i \xi_j^i \psi_j^i + q_j^i E_j^{6i} = 0 \tag{10}$$

Where A_1, A_2, A_3, A_4, C_1, C_2, C_3, C_4, B_j^{1i}, B_j^{2i}, B_j^{3i}, D_j^{1i}, D_j^{2i}, D_j^{3i}, E_j^{1i}, E_j^{2i}, E_j^{3i}, E_j^{4i}, E_j^{5i} and E_j^{6i} terms are given in Appendix A.

3. Motor Driven Mechanism Model

In traditional methodology design, when a motor is coupled to a mechanism, for a period of time varying torque is applied as an external load to the motor. This torque variation is undesirable when timing requirements are involved. Since the torque generated by the motor is proportional to the current and in order to introduce the motor parameters, in this study, the current is analyzed rather than the torque.

In this section, an ideal DC motor connected to a mechanical system is used to represent the electrical system. The driving motor model and the coupling equations between the electrical and the mechanical systems are presented.

3.1. Driving motor model

Fig.4 shows a schematic of a DC motor-gear system [8, 21, 22 and 23]. The voltage V is the input system, and the torque T_b, which is equal to the torque T defined in (9), is the output of the system. The ratio of the geared speed-reducer is

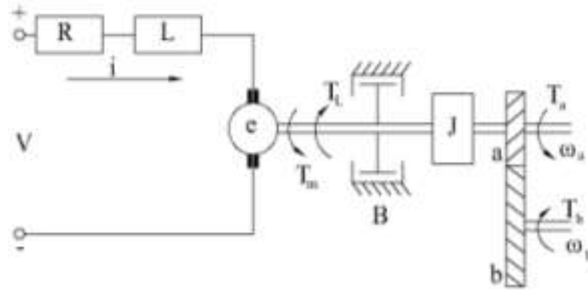

Fig. 4. Schematic of a DC motor-gear system

$$n = \frac{T_b}{T_a} = \frac{\omega_a}{\omega_b} \tag{11}$$

Using the Kirchhoff's voltage law yields

$$V = Ri(t) + L\frac{di(t)}{dt} + e \tag{12}$$

The equation of motion of the driving system is

$$T = n\left(T_m - T_L - B\omega_a - J\frac{d\omega_a}{dt}\right) \tag{13}$$

For a constant field current, the magnetic motor torque is

$$T_m = K_m i(t) \tag{14}$$

The constant electromotive force is

$$e = K_g\,\omega_a \tag{15}$$

Substituting (11), (14) and (15) in (12) and (13), we obtain

$$\frac{di(t)}{dt} = \frac{1}{L}\left(V - Ri(t) + nK_g\dot{\theta}_2\right) \tag{16}$$

and

$$T = nK_m i(t) - nT_L - n^2 B\dot{\theta}_2 - n^2 J\ddot{\theta}_2 \tag{17}$$

(16) and (17) represent the mathematical model of the DC motor. They give the current and motor torque variation with respect to time.

3.2. Motor-mechanism system model

For a mechatronic system, the torque produced by the motor in (17) should be equal to the torque needed for the mechanical system in (9). This equality is given by

$$C_1\ddot{\alpha}_2 + C_2\dot{\alpha}_2^2 + C_3\dot{\alpha}_2 + C_4 + \sum_{j=2}^{4}\sum_{i=1}^{s}(\ddot{q}_j^i D_j^{1i} + \dot{q}_j^i D_j^{2i} + q_j^i D_j^{3i}) = nK_m i(t) - nT_L - n^2 B\dot{\theta}_2 - n^2 J\ddot{\theta}_2 \tag{18}$$

Under the assumption that the crank rotates at a constant speed the second-order derivative terms of θ2 in (17) will disappear and all the derivative of θ2 in (16) and (18) are constant. Therefore, solving for i (t), (18) becomes

$$i(t)=\frac{1}{nK_m}\left[\begin{array}{l}C_1\ddot{\alpha}_2+C_2\dot{\alpha}_2^2+C_3\dot{\alpha}_2+C_4+\\ \sum_{j=2}^{4}\sum_{i=1}^{s}(\ddot{q}_j^iD_j^{1i}+\dot{q}_j^iD_j^{2i}+q_j^iD_j^{3i})+nT_L+n^2B\dot{\theta}_2\end{array}\right] \quad (19)$$

This equation gives the current required to maintain a constant crank speed at a steady state.

4. Simulation results and discussion

The object of this section is to highlight the effect of flexibility and clearance on the performance of the mechanism. In order to take into account the parameters of the electrical system, the current is analyzed rather than the torque. In the first case, a four-bar mechatronic system with perfect joints is studied with different modulus of elasticity. In the second case the clearance between the crank and the coupler r_2=0.2mm is added to the first case.

The mechanism and motor parameters used in these simulations are given by tables 1 and 2.

Table 1. Link parameters

L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	m_2 (Kg)
0.246	0.111	0.295	0.186	2.1
m_3 (Kg)	m_4 (Kg)	I_2 (Kg.m$_2$)	I_3 (Kg.m$_2$)	I_4 (Kg.m$_2$)
5.54	3.6	0.002	0.04	0.01

Table 2. Motor parameters

R (Ω)	L (H)	K_m (N m/a)	K_g (V s)	J (kg m$_2$)	T_L (Nm)	B (N m s)
0.4	0.05	0.678	0.678	0.056	0.0	0.226

4.1. Flexible four bar mechanism

Three cases are considered to investigate the elastic effect. The first case is the baseline case whereas in second and third cases the stiffness characteristics of the system are artificially modified by multiplying Young's modulus 510^{-2} and 10^3 fold respectively. The last case presents a "nearly rigid" system. The current required to maintain a constant crank speed is defined by

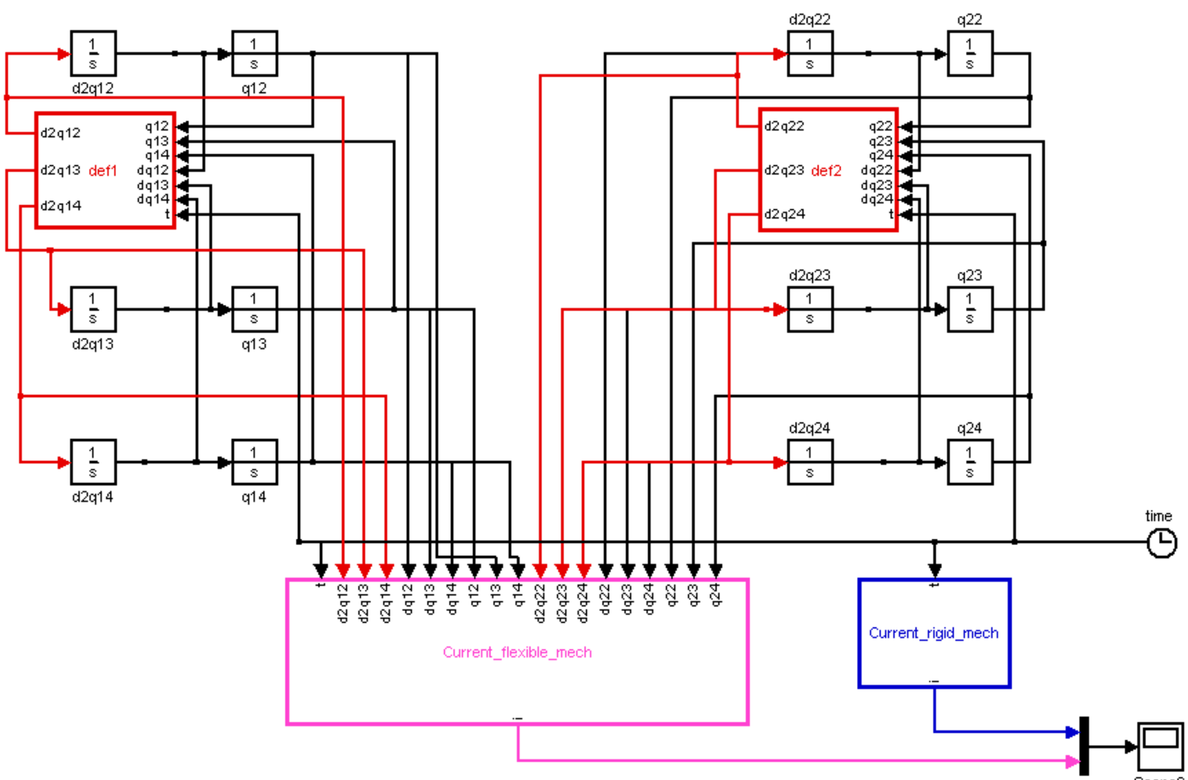

Fig. 5. Simulink model for flexible mechatronic mechanism.

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the two equations 16 and 19 in which r_2 and α_2 equal to 0. Solution of these equations is implemented by Simulink as shown in Fig.5. Fig.6 shows the current for the two cycles of motion for different cases.

We observe oscillations with respect to current of the rigid system. The current for the mechanism with flexible links includes an additional current represented by oscillations. The magnitude and the period of vibration increase as the young's modulus decreases. From this it reveals that the mechanical system applies more loads to the motor when his links are more elastic.

4.2. Flexible four bar mechanism with clearance

In this example, the clearance with the value 0.2 mm between the crank and the coupler is added to each case studied previously. The dynamic equations are given by the Eqs. 16 and 19.

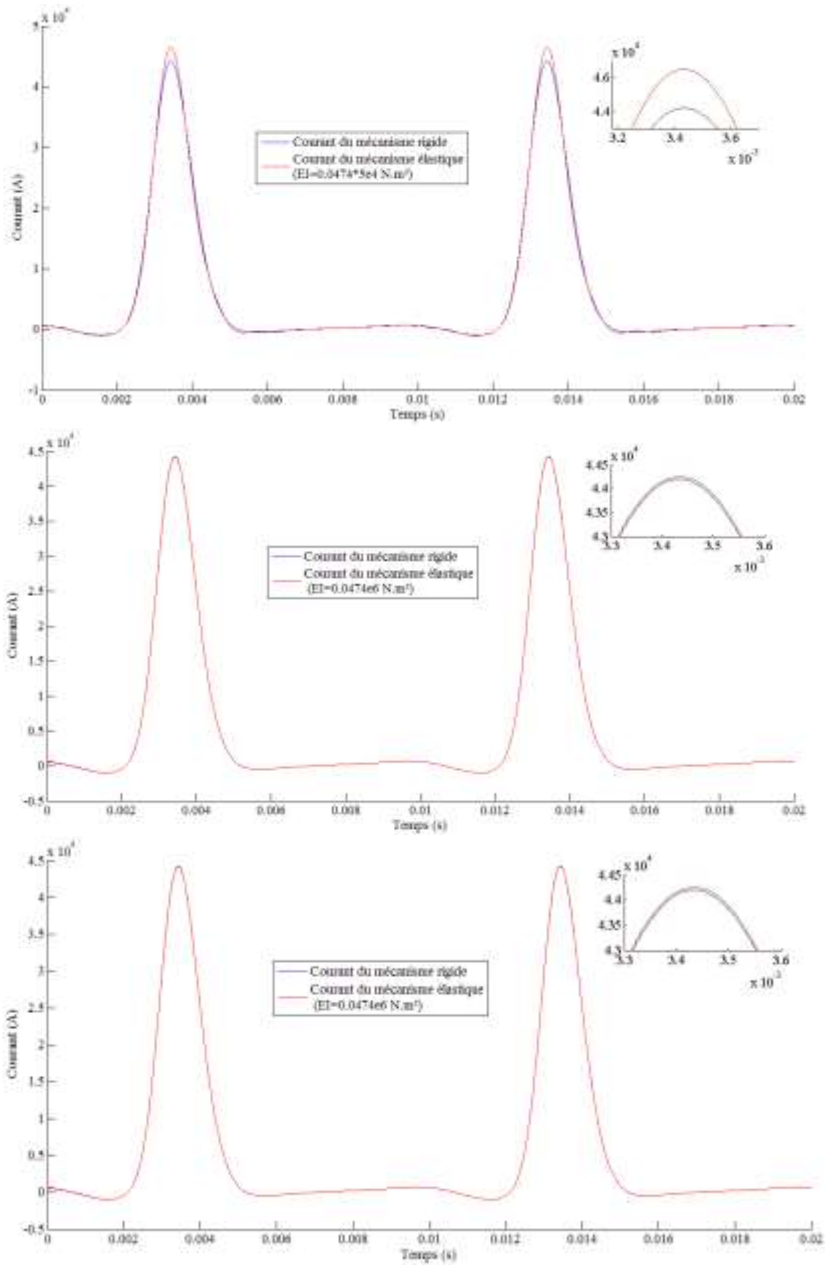

Fig. 6. Current for two cycles of motion for: EI=0.0474*5e^4, EI=0.0474e^6, EI=0.0474e^9.

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The Simulink model for the mechanism is shown in Fig.7. The blocs “Cr_clearance1” and “Subsystem1” are the Simulink sub models which allow calculating the currents of the rigid mechanism with clearance and flexible mechanism with clearance respectively. The bloc details of these sub models are shown in Fig.8 and Fig.9.

The current for two cycles of motion is shown in Fig.10.

The presence of clearance alters significantly the current required to drive the mechanism. The amplitude of the peaks decreases as the young’s modulus decreases (flexibility increase). It means that the flexibility has a role of suspension for the mechanism in the presence of clearance.

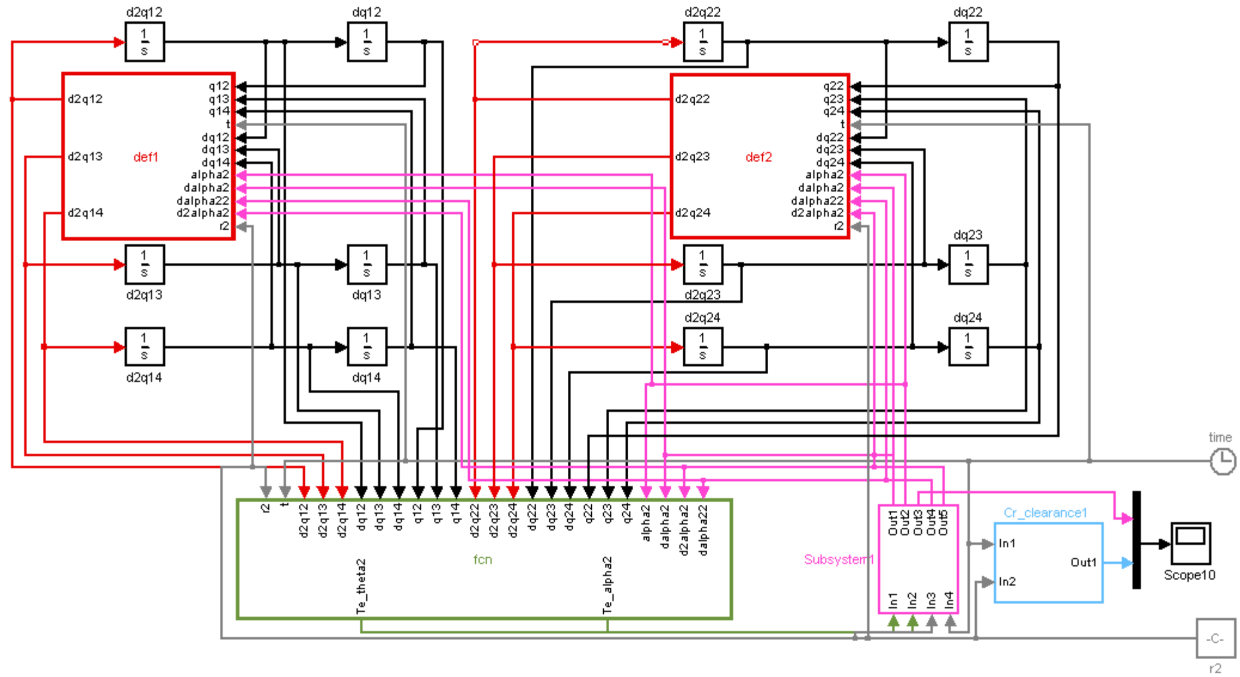

Fig. 7. Simulink model for flexible mechatronic mechanism with clearance.

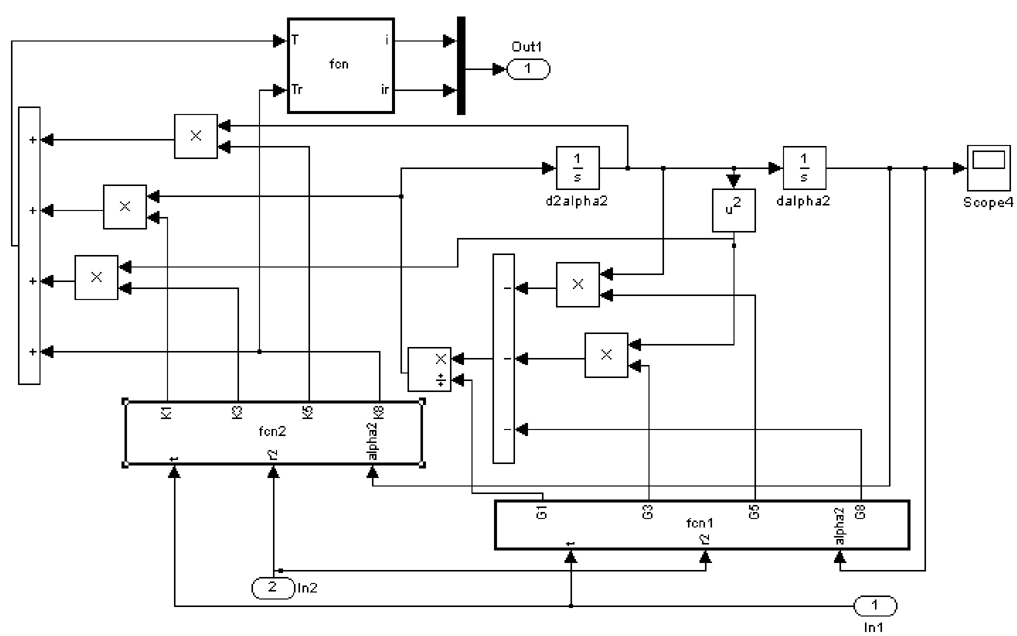

Fig. 8. Bloc detail of sub model “Cr_clearance1”.

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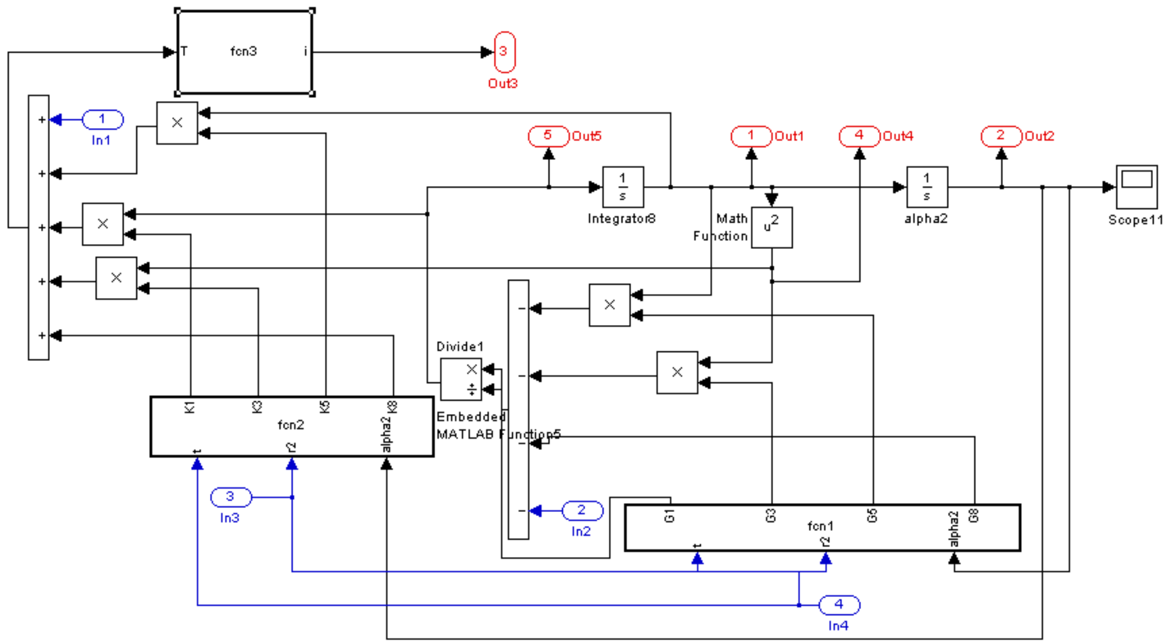

Fig. 9. Bloc detail of sub model "Subsystem1".

5. Conclusion

In this study, the subsystem interaction of a planar flexible four bar mechanism with joint clearance between crank and coupler driven by a controlled DC motor is considered.

The dynamic equations of the mechanism are derived by using Lagrange formulation. The current required for the mechanism to maintain a constant crank speed is defined analytically and then are implemented by using Simulink. The simulations are carried out for two cases; flexible mechanism without clearance and with different young's modulus and the same case with clearance. The results show that the flexible mechanical system without clearance applies a load to the electrical system. It's represented by the vibration of the current. The magnitude and the period of this vibration increase as the young's modulus decreases. It is also in the second case that the clearance alters greatly the current required to drive the rigid mechanism. The amplitude of the peaks is reduced when the flexibility is considered.

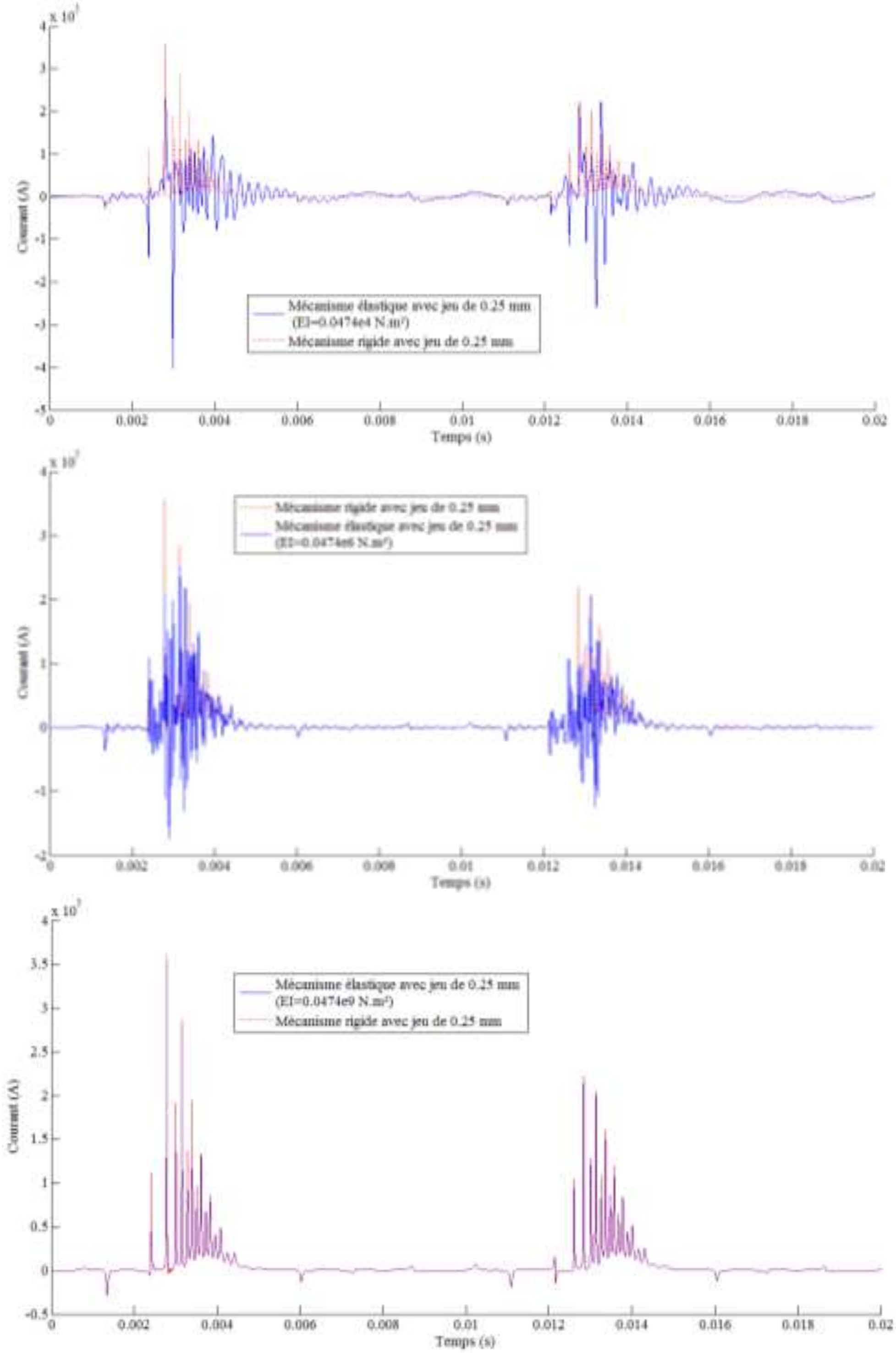

Fig. 10. Current for two cycles of motion.

Appendix

The terms A_1, A_2, A_3, A_4, C_1, C_2, C_3, C_4, B_j^{1i}, B_j^{2i}, B_j^{3i}, D_j^{1i}, D_j^{2i}, D_j^{3i}, E_j^{1i}, E_j^{2i}, E_j^{3i}, E_j^{4i}, E_j^{5i} and E_j^{6i} are given as follows :

$$\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{bmatrix} \left(\frac{\partial\theta_3}{\partial\alpha_2}\right)^2 & \left(\frac{\partial\theta_4}{\partial\alpha_2}\right)^2 & \left(\frac{\partial X_{G3}}{\partial\alpha_2}\right)^2 + \left(\frac{\partial Y_{G3}}{\partial\alpha_2}\right)^2 & \left(\frac{\partial X_{G4}}{\partial\alpha_2}\right)^2 + \left(\frac{\partial Y_{G4}}{\partial\alpha_2}\right)^2 \\ \frac{\partial^2\theta_3}{\partial\alpha_2{}^2}\frac{\partial\theta_3}{\partial\alpha_2} & \frac{\partial^2\theta_4}{\partial\alpha_2{}^2}\frac{\partial\theta_4}{\partial\alpha_2} & \frac{\partial^2 X_{G3}}{\partial\alpha_2{}^2}\frac{\partial X_{G3}}{\partial\alpha_2} + \frac{\partial^2 Y_{G3}}{\partial\alpha_2{}^2}\frac{\partial Y_{G3}}{\partial\alpha_2} & \frac{\partial^2 X_{G4}}{\partial\alpha_2{}^2}\frac{\partial X_{G4}}{\partial\alpha_2} + \frac{\partial^2 Y_{G4}}{\partial\alpha_2{}^2}\frac{\partial Y_{G4}}{\partial\alpha_2} \\ 2\omega_2\frac{\partial^2\theta_3}{\partial\alpha_2\partial\theta_2}\frac{\partial\theta_3}{\partial\alpha_2} & 2\omega_2\frac{\partial^2\theta_4}{\partial\alpha_2\partial\theta_2}\frac{\partial\theta_4}{\partial\alpha_2} & 2\omega_2\left(\frac{\partial^2 X_{G3}}{\partial\alpha_2\partial\theta_2}\frac{\partial X_{G3}}{\partial\alpha_2} + \frac{\partial^2 Y_{G3}}{\partial\alpha_2\partial\theta_2}\frac{\partial Y_{G3}}{\partial\alpha_2}\right) & 2\omega_2\left(\frac{\partial^2 X_{G4}}{\partial\alpha_2\partial\theta_2}\frac{\partial X_{G4}}{\partial\alpha_2} + \frac{\partial^2 Y_{G4}}{\partial\alpha_2\partial\theta_2}\frac{\partial Y_{G4}}{\partial\alpha_2}\right) \\ \omega_2^2\frac{\partial^2\theta_3}{\partial\theta_2{}^2}\frac{\partial\theta_3}{\partial\alpha_2} & \omega_2^2\frac{\partial^2\theta_4}{\partial\theta_2{}^2}\frac{\partial\theta_4}{\partial\alpha_2} & \omega_2^2\left(\frac{\partial^2 X_{G3}}{\partial\theta_2{}^2}\frac{\partial X_{G3}}{\partial\alpha_2} + \frac{\partial^2 Y_{G3}}{\partial\theta_2{}^2}\frac{\partial Y_{G3}}{\partial\alpha_2}\right) & \omega_2^2\left(\frac{\partial^2 X_{G4}}{\partial\theta_2{}^2}\frac{\partial X_{G4}}{\partial\alpha_2} + \frac{\partial^2 Y_{G4}}{\partial\theta_2{}^2}\frac{\partial Y_{G4}}{\partial\alpha_2}\right) \end{bmatrix} \begin{Bmatrix} I_3 \\ I_4 \\ m_3 \\ m_4 \end{Bmatrix}.$$

$$B_j^{1i} = \frac{\partial\theta_j}{\partial\alpha_2} m_j^{i\theta} + H_j^1 m_j^i$$

$$B_j^{2i} = -2m_j^i\left(\omega_2 H_j^2 + \dot{\alpha}_2 H_j^3\right)$$

$$B_j^{3i} = -m_j^i\left(\omega_2^2 H_j^4 + 2\omega_2\dot{\alpha}_2 H_j^5 + \ddot{\alpha} H_j^6 + \dot{\alpha}_2^2 H_j^7\right)$$

$$\begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{bmatrix} \frac{\partial\theta_3}{\partial\alpha_2}\frac{\partial\theta_3}{\partial\theta_2} & \frac{\partial\theta_4}{\partial\alpha_2}\frac{\partial\theta_4}{\partial\theta_2} & 0 & \frac{\partial X_{G3}}{\partial\alpha_2}\frac{\partial X_{G3}}{\partial\theta_2} + \frac{\partial Y_{G3}}{\partial\alpha_2}\frac{\partial Y_{G3}}{\partial\theta_2} & \frac{\partial X_{G4}}{\partial\alpha_2}\frac{\partial X_{G4}}{\partial\theta_2} + \frac{\partial Y_{G4}}{\partial\alpha_2}\frac{\partial Y_{G4}}{\partial\theta_2} \\ \frac{\partial^2\theta_3}{\partial\alpha_2^2}\frac{\partial\theta_3}{\partial\theta_2} & \frac{\partial^2\theta_4}{\partial\alpha_2^2}\frac{\partial\theta_4}{\partial\theta_2} & 0 & \frac{\partial^2 X_{G3}}{\partial\alpha_2^2}\frac{\partial X_{G3}}{\partial\theta_2} + \frac{\partial^2 Y_{G3}}{\partial\alpha_2^2}\frac{\partial Y_{G3}}{\partial\theta_2} & \frac{\partial^2 X_{G4}}{\partial\alpha_2^2}\frac{\partial X_{G4}}{\partial\theta_2} + \frac{\partial^2 Y_{G4}}{\partial\alpha_2^2}\frac{\partial Y_{G4}}{\partial\theta_2} \\ 2\omega_2\frac{\partial^2\theta_3}{\partial\alpha_2\partial\theta_2}\frac{\partial\theta_3}{\partial\theta_2} & 2\omega_2\frac{\partial^2\theta_4}{\partial\alpha_2\partial\theta_2}\frac{\partial\theta_4}{\partial\theta_2} & 0 & 2\omega_2\left(\frac{\partial^2 X_{G3}}{\partial\alpha_2\partial\theta_2}\frac{\partial X_{G3}}{\partial\theta_2} + \frac{\partial^2 Y_{G3}}{\partial\alpha_2\partial\theta_2}\frac{\partial Y_{G3}}{\partial\theta_2}\right) & 2\omega_2\left(\frac{\partial^2 X_{G4}}{\partial\alpha_2\partial\theta_2}\frac{\partial X_{G4}}{\partial\theta_2} + \frac{\partial^2 Y_{G4}}{\partial\alpha_2\partial\theta_2}\frac{\partial Y_{G4}}{\partial\theta_2}\right) \\ \omega_2^2\frac{\partial^2\theta_3}{\partial\theta_2{}^2}\frac{\partial\theta_3}{\partial\theta_2} & \omega_2^2\frac{\partial^2\theta_4}{\partial\theta_2{}^2}\frac{\partial\theta_4}{\partial\theta_2} & \omega_2^2\left(\frac{\partial^2 X_{G2}}{\partial\theta_2{}^2}\frac{\partial X_{G2}}{\partial\theta_2} + \frac{\partial^2 Y_{G2}}{\partial\theta_2{}^2}\frac{\partial Y_{G2}}{\partial\theta_2}\right) + g\frac{\partial Y_{G2}}{\partial\theta_2} & \omega_2^2\left(\frac{\partial^2 X_{G3}}{\partial\theta_2{}^2}\frac{\partial X_{G3}}{\partial\theta_2} + \frac{\partial^2 Y_{G3}}{\partial\theta_2{}^2}\frac{\partial Y_{G3}}{\partial\theta_2}\right) + g\frac{\partial Y_{G3}}{\partial\theta_2} & \omega_2^2\left(\frac{\partial^2 X_{G4}}{\partial\theta_2{}^2}\frac{\partial X_{G4}}{\partial\theta_2} + \frac{\partial^2 Y_{G4}}{\partial\theta_2{}^2}\frac{\partial Y_{G4}}{\partial\theta_2}\right) + g\frac{\partial Y_{G4}}{\partial\theta_2} \end{bmatrix} \begin{Bmatrix} I_3 \\ I_4 \\ m_2 \\ m_3 \\ m_4 \end{Bmatrix}$$

$$D_j^{1i} = \frac{\partial\theta_j}{\partial\theta_2} m_j^{i\theta} + G_j^1 m_j^i$$

$$D_j^{2i} = -2m_j^i\left(\omega_2 G_j^2 + \dot{\alpha}_2 G_j^3\right)$$

$$D_j^{3i} = -m_j^i\left(\omega_2^2 G_j^4 + g\frac{\partial\theta_j}{\partial\theta_2}\sin\theta_j + 2\omega_2\dot{\alpha}_2 G_j^5 + \ddot{\alpha}_2 G_j^6 + \dot{\alpha}_2^2 G_j^7\right)$$

$$\begin{Bmatrix} E_j^1 \\ E_j^2 \\ E_j^3 \\ E_j^4 \\ E_j^5 \\ E_j^6 \end{Bmatrix} = \frac{-1}{m_j^{ii}} \begin{bmatrix} 0 & \frac{\partial\theta_j}{\partial\alpha_2} & \frac{\partial Y_{Gj}}{\partial\alpha_2} & \frac{\partial X_{Gj}}{\partial\alpha_2} \\ 0 & \frac{\partial^2\theta_j}{\partial\alpha_2{}^2} & \frac{\partial^2 Y_{Gj}}{\partial\alpha_2{}^2} & \frac{\partial^2 X_{Gj}}{\partial\alpha_2{}^2} \\ 0 & 2\omega_2\frac{\partial^2\theta_j}{\partial\theta_2\partial\alpha_2} & 2\omega_2\frac{\partial^2 Y_{Gj}}{\partial\theta_2\partial\alpha_2} & 2\omega_2\frac{\partial^2 X_{Gj}}{\partial\theta_2\partial\alpha_2} \\ 0 & \omega_2^2\frac{\partial^2\theta_j}{\partial\theta_2^2} & \omega_2^2\frac{\partial^2 Y_{Gj}}{\partial\theta_2^2} & \omega_2^2\frac{\partial^2 X_{Gj}}{\partial\theta_2^2} \\ 1 & 0 & 0 & 0 \\ P1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} m_j^{ii} \\ m_j^{i\theta} \\ m_j^i\cos\theta_j \\ -m_j^i\sin\theta_j \end{Bmatrix}$$

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Where:

$$\begin{Bmatrix} H_j^1 \\ H_j^2 \\ H_j^3 \\ H_j^4 \\ H_j^5 \\ H_j^6 \\ H_j^7 \end{Bmatrix} = \begin{bmatrix} \frac{\partial Y_{Gj}}{\partial \alpha_2} & -\frac{\partial X_{Gj}}{\partial \alpha_2} \\ \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial X_{Gj}}{\partial \alpha_2} & \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial Y_{Gj}}{\partial \alpha_2} \\ \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial X_{Gj}}{\partial \alpha_2} & \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial Y_{Gj}}{\partial \alpha_2} \\ \frac{\partial^2 \theta_j}{\partial \theta_2^{\,2}}\frac{\partial X_{Gj}}{\partial \alpha_2} + \left(\frac{\partial \theta_j}{\partial \theta_2}\right)^2 \frac{\partial Y_{Gj}}{\partial \alpha_2} + \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial^2 X_{Gj}}{\partial \theta_2^2} & \frac{\partial^2 \theta_j}{\partial \theta_2^{\,2}}\frac{\partial Y_{Gj}}{\partial \alpha_2} - \left(\frac{\partial \theta_j}{\partial \theta_2}\right)^2 \frac{\partial X_{Gj}}{\partial \alpha_2} + \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial^2 Y_{Gj}}{\partial \theta_2^2} + g\frac{\partial \theta_j}{\partial \alpha_2} \\ \frac{\partial^2 \theta_j}{\partial \theta_2 \partial \alpha_2}\frac{\partial X_{Gj}}{\partial \alpha_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial Y_{Gj}}{\partial \alpha_2} + \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial^2 X_{Gj}}{\partial \theta_2 \partial \alpha_2} & \frac{\partial^2 \theta_j}{\partial \theta_2 \partial \alpha_2}\frac{\partial Y_{Gj}}{\partial \alpha_2} - \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial X_{Gj}}{\partial \alpha_2} + \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial^2 Y_{Gj}}{\partial \theta_2 \partial \alpha_2} \\ 2\frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial X_{Gj}}{\partial \alpha_2} & 2\frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial Y_{Gj}}{\partial \alpha_2} \\ \frac{\partial^2 \theta_j}{\partial \alpha_2^{\,2}}\frac{\partial X_{Gj}}{\partial \alpha_2} + \left(\frac{\partial \theta_j}{\partial \alpha_2}\right)^2 \frac{\partial Y_{Gj}}{\partial \alpha_2} + \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial^2 X_{Gj}}{\partial \alpha_2^2} & \frac{\partial^2 \theta_j}{\partial \alpha_2^{\,2}}\frac{\partial Y_{Gj}}{\partial \alpha_2} - \left(\frac{\partial \theta_j}{\partial \alpha_2}\right)^2 \frac{\partial X_{Gj}}{\partial \alpha_2} + \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial^2 Y_{Gj}}{\partial \alpha_2^2} \end{bmatrix} \begin{Bmatrix} \cos\theta_j \\ \sin\theta_j \end{Bmatrix}.$$

$$\begin{Bmatrix} G_j^1 \\ G_j^2 \\ G_j^3 \\ G_j^4 \\ G_j^5 \\ G_j^6 \\ G_j^7 \end{Bmatrix} = \begin{bmatrix} \frac{\partial Y_{Gj}}{\partial \theta_2} & -\frac{\partial X_{Gj}}{\partial \theta_2} \\ \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial X_{Gj}}{\partial \theta_2} & \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial Y_{Gj}}{\partial \theta_2} \\ \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial X_{Gj}}{\partial \theta_2} & \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial Y_{Gj}}{\partial \theta_2} \\ \frac{\partial^2 \theta_j}{\partial \theta_2^{\,2}}\frac{\partial X_{Gj}}{\partial \theta_2} + \left(\frac{\partial \theta_j}{\partial \theta_2}\right)^2 \frac{\partial Y_{Gj}}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial^2 X_{Gj}}{\partial \theta_2^2} & \frac{\partial^2 \theta_j}{\partial \theta_2^{\,2}}\frac{\partial Y_{Gj}}{\partial \theta_2} - \left(\frac{\partial \theta_j}{\partial \theta_2}\right)^2 \frac{\partial X_{Gj}}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial^2 Y_{Gj}}{\partial \theta_2^2} \\ \frac{\partial^2 \theta_j}{\partial \theta_2 \partial \alpha_2}\frac{\partial X_{Gj}}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial Y_{Gj}}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial^2 X_{Gj}}{\partial \theta_2 \partial \alpha_2} & \frac{\partial^2 \theta_j}{\partial \theta_2 \partial \alpha_2}\frac{\partial Y_{Gj}}{\partial \theta_2} - \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial X_{Gj}}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial^2 Y_{Gj}}{\partial \theta_2 \partial \alpha_2} \\ \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial X_{Gj}}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial X_{Gj}}{\partial \alpha_2} & \frac{\partial \theta_j}{\partial \alpha_2}\frac{\partial Y}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial Y_{Gj}}{\partial \alpha_2} \\ \frac{\partial^2 \theta_j}{\partial \alpha_2^{\,2}}\frac{\partial X_{Gj}}{\partial \theta_2} + \left(\frac{\partial \theta_j}{\partial \alpha_2}\right)^2 \frac{\partial Y_{Gj}}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial^2 X_{Gj}}{\partial \alpha_2^2} & \frac{\partial^2 \theta_j}{\partial \alpha_2^{\,2}}\frac{\partial Y_{Gj}}{\partial \theta_2} - \left(\frac{\partial \theta_j}{\partial \alpha_2}\right)^2 \frac{\partial X_{Gj}}{\partial \theta_2} + \frac{\partial \theta_j}{\partial \theta_2}\frac{\partial^2 Y_{Gj}}{\partial \alpha_2^2} \end{bmatrix} \begin{Bmatrix} \cos\theta_j \\ \sin\theta_j \end{Bmatrix}.$$

$$P1 = -\left[\omega_2^2\left(\frac{\partial \theta_j}{\partial \theta_2}\right)^2 + 2\omega_2\dot{\alpha}_2\frac{\partial \theta_j}{\partial \theta_2}\frac{\partial \theta_j}{\partial \alpha_2} + \dot{\alpha}_2^2\left(\frac{\partial \theta_j}{\partial \alpha_2}\right)^2 - \psi_j^{i\,2}\right]$$

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