Mechanical Error Analysis of Disc Cam Mechanisms

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Abstract: This paper presents a novel and simple approach to analyze the mechanical error in disk cam mechanisms with a roller follower. The error of the follower motion due to tolerances is determined geometrically by employing the concept of the equivalent linkage and smaller displacements diagram. The offset translating roller follower and the oscillating roller follower cases are given to illustrate the proposed method. The tolerance analysis results show that the errors in disk cam mechanism are closely related to the value of the pressure angle and the location of the curvature center of the cam profile is not essential. The resulting error due to the radial dimension error of the cam profile has relatively smaller variation owing to the counteraction of the shift angle. In the worst case, owing to the combined effect of various design parameters, the accuracy of the follower motion may degrade considerably.

Keywords: Tolerance analysis, disc cam mechanism, equivalent linkage, smaller displacements diagram.

1. Introduction

The cam mechanism can provide a simple, compact and reliable means of motion control in machinery. In high-speed machinery, even small errors in cam contour may produce excessive noise, wear and vibrations [1-3]. The expected kinematic accuracy of the follower motion, as well as the dynamic performance, of the cam mechanism may degrade considerably. To maintain reasonably acceptable performance of cam mechanisms, sufficient accuracy of the cam contour and consequent close tolerances should be considered in the design phase.

For precision cam mechanisms, designers should specify the tolerances of the cam profile and that of each design parameter at the largest (or optimal) values to meet the operating or functional considerations of the follower output. Therefore, theoretical correlation between tolerance level of each design parameter and the follower motion deviation must be established. That is, the tolerance analysis, also known as the mechanical error analysis [4,5], is one of the key elements of optimal tolerance design for cam mechanisms.

There have been some approaches presented for the tolerance analysis of disk cam mechanisms [6-12]. These approaches can provide a valuable source of idea for this analysis. However, they cannot provide a simple means for predicting the mechanical errors of cam mechanisms. Thus, the purpose of this paper is to offer a simple geometrical model to identify the output position error of cam mechanisms. Therefore, the present paper is organised as follows. Section 2 gives parametric expression for the cam profile. In section 3 geometrical model to calculate the error of the follower motion in cam mechanisms with roller follower is presented. Radial error of the cam profile is given in section 4. An example is presented in section 5 to illustrate the proposed method. Finally, conclusion is outlined in section 6.

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2. Parametric expressions for the cam profile

In order to find the relation between the manufacturing error of a machined cam profile and the corresponding output deviation of its follower, the analytical expressions for the theoretical cam profile should be derived first. The profile of a disc cam can be determined through the use of velocity instant centres. For quick review and easy reference, the analytical approach developed by the author in reference [13] is provided below. Here, disc cams with a roller follower, an offset translating roller follower or an oscillating roller follower, are demonstrated.

2.1. Cams with an offset translating roller follower

Fig. 1 shows a disc cam mechanism with an offset translating roller follower. Setting up a Cartesian coordinate system (X, Y) fixed on the cam, and with its origin at the fixed pivot O_2, the cam profile coordinates may be expressed in terms of the cam rotation angle θ, which is measured against the direction of cam rotation from the reference radial to cam centre parallel to roller translation. For simplicity, in the following, the ground link will be consistently numbered as 1, the cam as 2, and the follower as 4. In addition, the coupler in an equivalent linkage of a cam mechanism will be numbered as 3 in the following sections. By labelling instant centre I_{24} as Q and O_2Q = q, the vectors of the cam profile coordinates are

$$O_2A = O_2E + EC + CA \qquad (1)$$

$$= \begin{Bmatrix} L(\theta)\cos\theta - e\sin\theta - r_f\cos(\theta-\phi) \\ L(\theta)\sin\theta + e\cos\theta - r_f\sin(\theta-\phi) \end{Bmatrix} \qquad (2)$$

Where

$$\phi = \tan^{-1}\left[\frac{q-e}{L(\theta)}\right] \qquad (3)$$

$$q = \frac{dL(\theta)}{d\theta} \qquad (4)$$

in which, point C is the roller center and $L(\theta)$ is the linear displacement function of the follower:

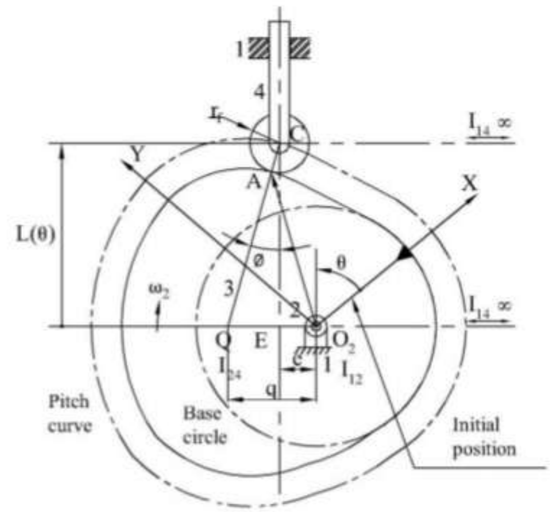

Fig. 1. Disc cam with offset translating roller follower

$$L(\theta) = \sqrt{(r_b + r_f)^2 - e^2} + S(\theta) \qquad (5)$$

Where r_b is the radius of the base circle, r_f is the radius of the follower roller, e is the offset and $S(\theta)$ is the specified linear motion program of the follower (i.e. the so-called cam displacement function [1,2]). (Since the cam is to rotate clockwise, the quantity e is negative if the offset is to the right; in the position shown it is positive). Also, φ is the pressure angle of the cam mechanism. The common normal at the contact point A must always pass through point Q, A and the roller center C.

2.2 Disk cam with an oscillating roller follower

Fig. 2 shows a disk cam mechanism with an oscillating roller follower. In this case, f represents the distance from the cam centre to the follower pivot point and l represents the arm length of the follower. Setting up a Cartesian coordinate system X–Y fixed on the cam and with its origin at the fixed pivot O_2, the cam profile coordinates may be expressed in terms of θ, which is measured against the direction of cam rotation from the reference radial on cam to line O_2O_3. The cam is to rotate clockwise with a constant angular velocity of ω_2 rad/s. By labelling instant centre I_{23} as Q and O_2Q = q, the parametric vector equations of the theoretical cam profile coordinates are [10,13],

$$O_2A = O_2Q + QA \tag{6}$$

$$= \begin{Bmatrix} (QC - r_f)\cos(\theta+\alpha) - q\cos\theta \\ (QC - r_f)\sin(\theta+\alpha) - q\sin\theta \end{Bmatrix} \tag{7}$$

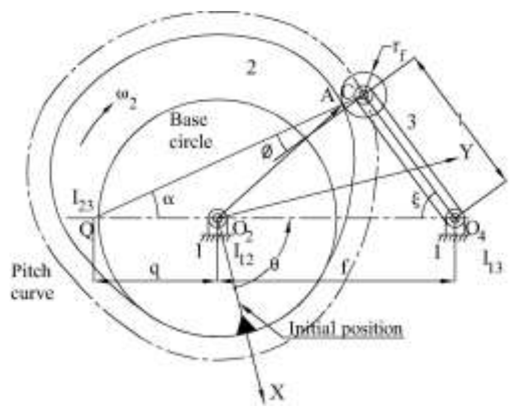

Fig. 2. Disc cam with an oscillating roller follower

Where

$$q = \frac{f\,\frac{d\xi(\theta)}{d\theta}}{1 - \frac{d\xi(\theta)}{d\theta}} \tag{8}$$

$$QC = \sqrt{l^2 + (f+q)^2 - 2l(f+q)\cos\xi(\theta)} \tag{9}$$

$$\alpha = \sin^{-1}\left[\frac{l\sin\xi(\theta)}{QC}\right] \tag{10}$$

In which, point C is the roller center and $\xi(\theta)$ is the angular displacement function of the follower [10,13]

$$\xi(\theta) = \cos^{-1}\left[\frac{l^2 + f^2 - (r_b + r_f)^2}{2lf}\right] + S(\theta) \tag{11}$$

Where r_b is the radius of the base circle, rf is the radius of the follower roller and $S(\theta)$ is the specified angular motion program of the follower (i.e., the so-called cam displacement function). Also, the pressure angle φ can be expressed as

$$\phi = 90^\circ - \alpha - \xi(\theta) \tag{12}$$

3. Geometrical method

This method employs the concept of equivalent linkage and smaller displacements diagram for analysing the mechanical error of the planar linkage. The representation of smaller displacements diagram, applied to the transformed equivalent linkage, is similar to the kinematical diagram [14].

3.1. Kinematical diagram

Consider two points A and B in the rigid body S (fig. 3) and the application ξ

$$\forall P \in S \subset R^2$$

$$\xi : P \rightarrow p \in R^2$$

where P is a point of a real plan and $\overrightarrow{op} = \vec{V}(P)$ is the velocity vector of P. The image of the fixed-point O (velocity equal zero) of the rigid body S is the point o.

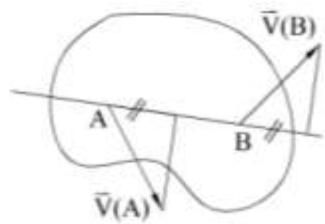

Fig. 3. Vector velocities of point in rigid body

The point a (respectively b) denotes the image of A (respectively B). The property of the Equiprojectivity of velocity fields gives:

$$(\vec{V}(B) - \vec{V}(A)).\overrightarrow{AB} = 0 \tag{13}$$

$$(\overrightarrow{ob} - \overrightarrow{oa}).\overrightarrow{AB} = 0 \tag{14}$$

$$\overrightarrow{ab}.\overrightarrow{AB} = 0 \tag{15}$$

From the expression (15), we can deduce that the direction of the image (ab) have to be perpendicular to the direction of line (AB). Thus, the kinematical diagram can be given by Fig. 4.

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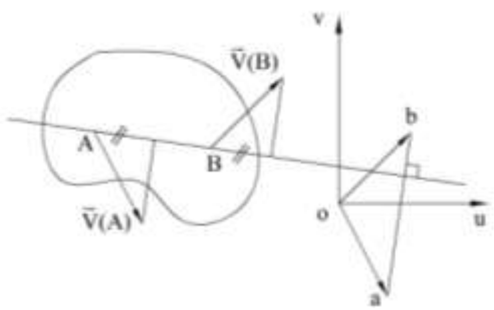

Fig. 4. Kinematical diagram

3.2. Cams with an offset translating roller follower

For the cam mechanism shown in Fig. 5a, let point K be the centre of curvature of the cam in contact with the follower. Its equivalent linkage is the slider–crank linkage shown in Fig. 5b, in which the coupler (link 3) of the linkage connects the centre of curvature of the cam K, and the roller centre C.

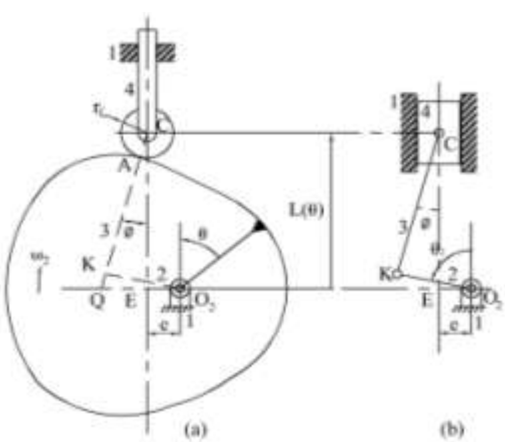

Fig. 5. Cam mechanism and its equivalent linkage

The actual profile of a machined cam may slightly deviate from the theoretical, and an error in the follower motion will thus be produced. Since the instantaneous kinematic characteristics of a cam mechanism are identical to those of its equivalent linkage [4], the mechanical error analysis of a cam mechanism can be performed through the aid of its equivalent linkage. In other words, if the profile error in the normal direction of a machined cam, Δn, equals the coupler-length error of the equivalent linkage, Δl_3, their output links will have the same motion deviations. As a result, the procedure developed by [14] can be applied to calculate the output motion deviation of the equivalent linkage.

3.2.1. Error due to the error of the cam profile Δl_3

The substituted equivalent linkage is shown in Fig. 6a. The point K is assumed to move along the axis of the element 3. Therefore, an additional slider 5 shown in dashed line is introduced. It is susceptible to slide along the axis (CK). Assuming that the slider displacement is $+\Delta l_3$, the position error can be calculated by using the smaller displacements diagram shown in Fig. 6b.

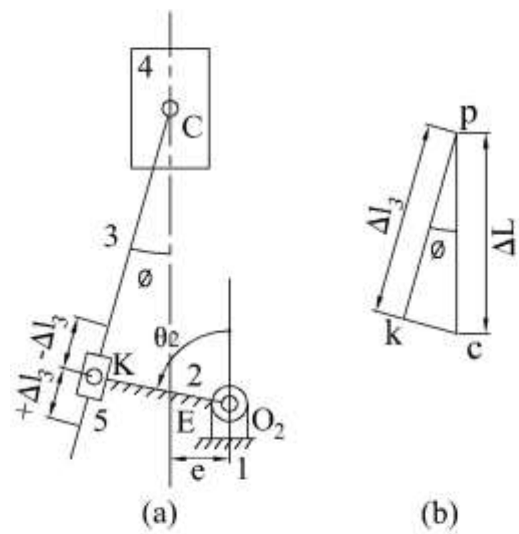

Fig. 6. Substituted equivalent linkage and its smaller displacements diagram

From triangle pkc and sine law, the mechanical error at the output due to the error of the cam profile Δl_3 can be expressed as

$$\Delta L = \frac{\Delta l_3}{\cos \phi} \quad (16)$$

Since the output error ΔL is identical to the

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follower motion error ΔS,

$$\Delta S_n = \Delta L = \frac{\Delta l_3}{\cos\phi} \qquad (17)$$

3.2.2. Error due to the radius error of the follower roller Δr_f

The substituted equivalent linkage is shown in Fig. 7a. The additional slider 5 shown in dashed line is susceptible to slide along the axis (CK). Assuming that the slider displacement is $+\Delta r_f$, the position error can be calculated by using smaller displacements diagram shown in Fig. 7b.

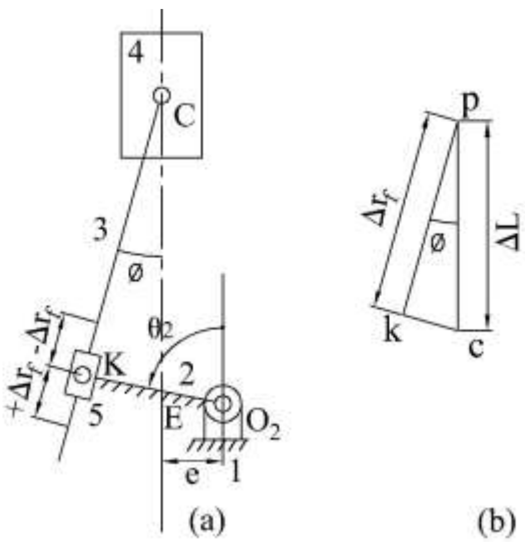

Fig. 7. Substituted equivalent linkage and its smaller displacements diagram

From triangle pkc and sine law, the mechanical error at the output due to the error of the radius error of the follower roller Δr_f can be expressed as

$$\Delta L = \frac{\Delta r_f}{\cos\phi} \qquad (18)$$

Since the output error ΔL is identical to the follower motion error ΔS,

$$\Delta S_{rf} = \Delta L = \frac{\Delta r_f}{\cos\phi} \qquad (19)$$

Equations (17) and (19) indicate that, the pressure angle has a significant effect on the resulting errors. In addition, it is interesting to note that, the parameters $l_2 = O_2K$ and $l_3 = KC$ are not actually involved. In other words, locating the curvature centre of the cam profile in the analysis process is not really essential, and this fact makes the analysis easier to perform.

3.3. Cams with an oscillating roller follower

Fig. 8 shows the equivalent linkage of the cam mechanism with an oscillating roller follower. The analysis presented in this section can be applied to investigate the link-length effect on the performance of four bar mechanism studied in paper [15].

3.3.1. Error due to the error of the cam profile Δl_3

The substituted equivalent linkage is shown in Fig. 9a. The point K is assumed to move along the axis of the element 3. Therefore, an additional slider 5 shown in dashed line is introduced. It is susceptible to slide along the axis (CK).

For small values of the errors Δl_3 and $\Delta\theta_4$ the motion of point C can be considered linear and perpendicular to the arm of the follower and thus

$$\Delta C = -l\Delta\theta_4 \qquad (20)$$

Assuming that slider displacement is $+\Delta l_3$, the position error can be calculated by using smaller displacements diagram shown in Fig. 9b.

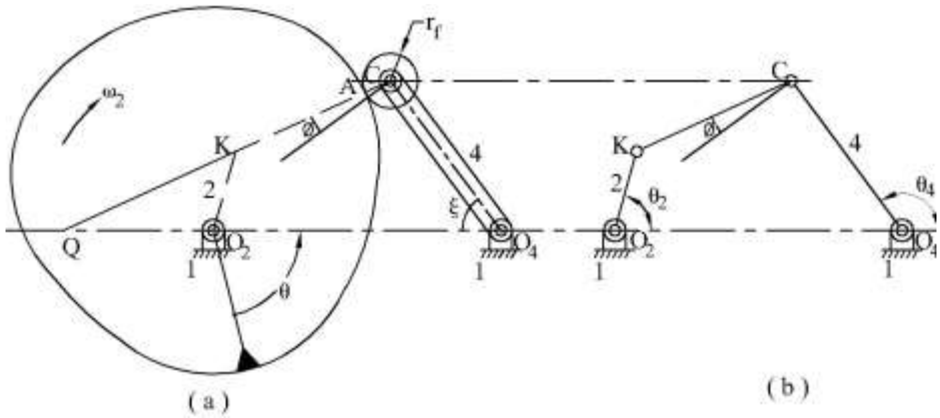

Fig. 8. Cam mechanism and its equivalent linkage

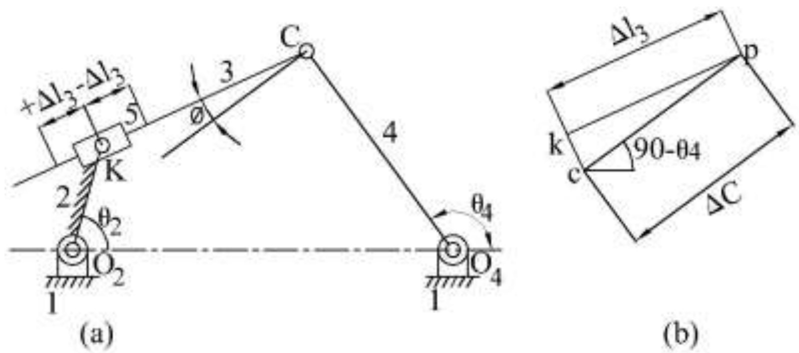

Fig. 9. Substituted equivalent linkage and its smaller displacements diagram

Sine law to triangle pkc yields

$$\Delta C = \frac{\Delta l_3}{\cos\phi} \quad (21)$$

Recall from Fig. 8 that $\xi(\theta) = 180° - \theta_4$,

and thus $\Delta S = \Delta\xi = -\Delta\theta_4$. The final result is

$$\Delta S_n = -\Delta\theta_4 = \frac{\Delta l_3}{l\cos\phi} \quad (22)$$

3.3.2. Error due to the radius error of the follower Δr_f

The substituted equivalent linkage is shown in Fig. 10a. Assuming that the slider displacement is $+\Delta r_f$, the position error can be calculated by using kinematic diagram shown in Fig.10b.

Sine law to triangle pkc yields

$$\Delta C = \frac{\Delta r_f}{\cos\phi} \quad (23)$$

For small angular displacement $\Delta\theta_4$, ΔC can be expressed as

$$\Delta C = -l\Delta\theta_4 \quad (24)$$

The final result is

$$\Delta S_{rf} = \Delta\xi = -\Delta\theta_4 = \frac{\Delta r_f}{l\cos\phi} \quad (25)$$

Equations (22) and (25) indicate that the pressure angle has a significant effect on the resulting errors. In addition, it is interesting to note that, the parameters l_2 and l_3 are not actually involved. In other words, locating the curvature center of the cam profile in the analysis process is not really essential, and this fact makes the analysis easier to perform.

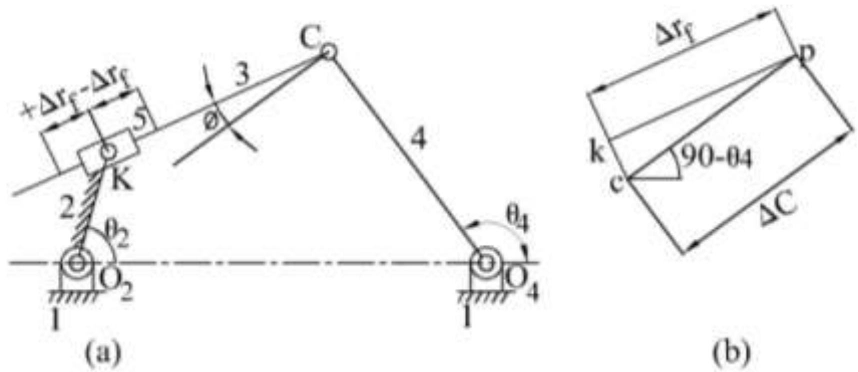

Fig. 10. Substituted equivalent linkage and its smaller displacements diagram

4. Radial dimension error of the cam profile

In practice, the profile accuracy of a machined cam may be controlled through a properly specified tolerance of the radial dimension of the actual cam profile. In other words, the radial dimensions of the actual cam profile must lie within a specified zone along the ideal profile. However, the actual profile of a machined cam may deviate from that of the theoretical one, and thus undesirable motion error of the follower will result.

Fig. 11 shows a cam in contact with its roller follower, in which the theoretical cam profile is shown by the solid line and the actual cam profile by the dashed line, and the profile deviations exaggerated for clarity. The theoretical contact point is designated by A, and its normal to the cam profile intersects the actual cam profile at point A_n; line O_2A intersects the actual cam profile at point Ar. For a sufficiently small value of normal-direction profile error AA_n, since A_nA_r will be tangent to the actual cam profile and A_nA will be normal to the profile, triangle AA_nA_r can be considered as a right-angled triangle, and thus

$$\Delta n \approx \Delta r \cos \lambda \qquad (26)$$

Where $\Delta n = AA_n$, $\Delta r = AA_r$

and $\lambda = \angle A_r AA_n$. In addition, the common normal at contact point A pass through point Q, and so

$$\lambda = \angle O_2 AQ \qquad (27)$$

From triangle O2AQ

$$\lambda = \sin^{-1}\left(\frac{QA \times O_2 A}{\|QA\|.\|O_2 A\|}\right) \qquad (28)$$

This is a general expression for the shift angle λ and is applicable to all types of disc cam mechanisms.

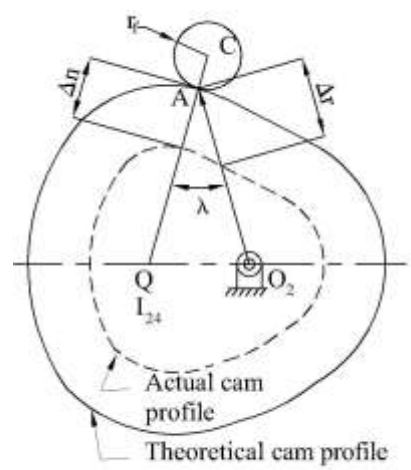

Fig. 11. Actual cam profile and theoretical cam profile

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For the cam mechanism with an offset translating roller follower, from equations (17) and (26), the mechanical error is expressed as

$$\Delta S_r = \Delta L \approx \frac{\Delta r \cos\lambda}{\cos\phi} \quad (29)$$

This shows how the radial dimension error of the cam profile affects the motion deviation of the translating follower. Note that the effect of the pressure angle ϕ on the resulting error is here counteracted by the shift angle λ with the form $\cos\lambda$.

Similarly, for the cam mechanism with an oscillating roller follower, from equations (22) and (26), the mechanical error is expressed as

$$\Delta S_r = -\Delta\theta_4 \approx \frac{\Delta r \cos\lambda}{l\cos\phi} \quad (30)$$

This shows how the radial dimension error of the cam profile affects the motion deviation of the oscillating follower. Similarly, in this case, the effect of the pressure angle is counteracted by the shift angle.

5. Example

The method presented above will be illustrated by the following example [9]:

A cam system requires the offset translating roller follower to rise 24 mm with cycloidal motion while the cam rotates clockwise from 0 to 120°, dwell for the next 50°, return with cycloidal motion for 120° cam rotation, and dwell for the remaining 70°. The offset e is 12 mm. The radii of the base circle and the follower roller are 40 and 10 mm respectively. The cam profile, which has a maximum radial dimension of 63.525 mm, is shown in Fig. 1. For a tolerance grade of IT6, the cam profile may have a deviation of $\Delta r = 19\mu m$ and the follower may have a deviation of $\Delta r_f = 9\mu m$. The motion of the follower will then have deviations resulting from them, ΔS_r due to Δr and ΔS_{rf} due to Δr_f. The worst case deviation of the follower motion will be [4],

$$\Delta S_{wor}(\theta) = |\Delta S_r(\theta)| + |\Delta S_{rf}(\theta)| \quad (31)$$

The maximum expected deviation of the follower motion will be [4],

$$\Delta S_{rms} = \sqrt{\Delta S_r^2 + \Delta S_{rf}^2} \quad (32)$$

which is often referred to as the root mean square value.

All functions that might be of interest are shown in Fig. 12. Figure 12a shows that the pressure angle Φ and the shift angle λ have quite similar trends. Therefore, the variation of (1/cos Φ) is flattened by cosλ. As shown in Figs 12.a and b, since $0.8 \le \cos\lambda/\cos\varphi \le 1.08$, the magnitude of ΔS_r and ΔS_{rf} has only slight variation. The extreme value of ΔS_{wor} occurs at $\theta = 260°$. In addition, ΔS_{wor} has an extreme value of 29.8 μm. From the viewpoint of the position accuracy of the follower motion, for a total follower travel of 24 mm, a position deviation of $\Delta S_{wor} = 29.8\mu m$ implies a lower accuracy. That is, if the worst situation occurs, the follower motion will have a degraded accuracy ranging from IT7 (21 μm) to IT8 (33 μm).

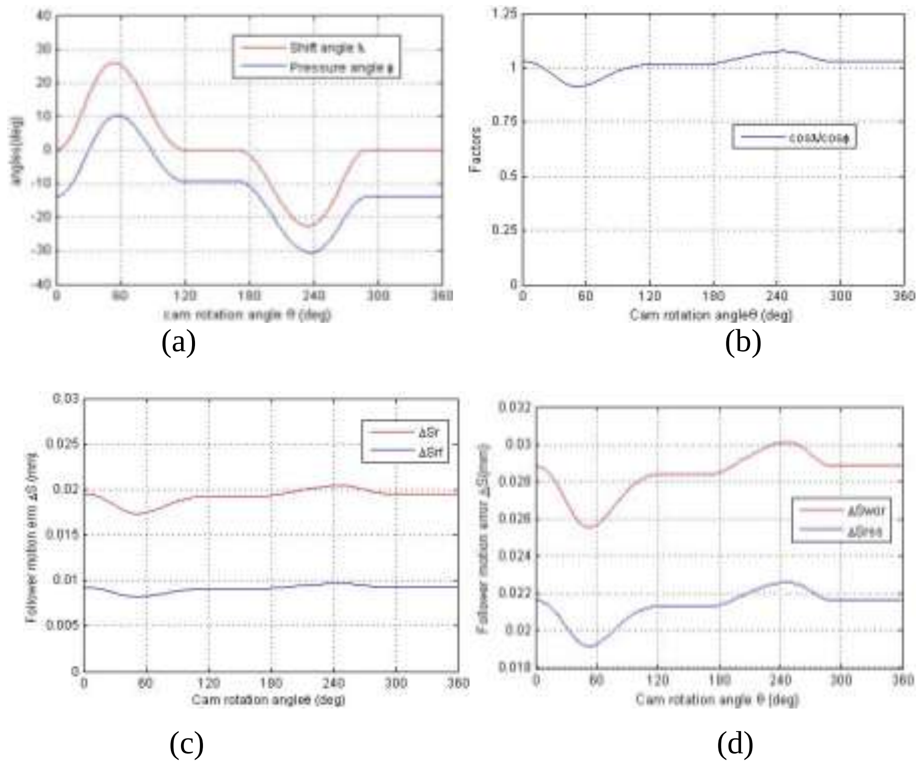

Fig. 12. Mechanical error of cam mechanism with an offset translating roller.

6. Conclusion

This paper presents a geometrical method to analyze tolerances in disc cam mechanism with a roller follower. This method offers a simple and effective model to determinate the error of the follower motion due to the variation in each design parameter. It proves that the location of the curvature center of the cam profile is not essential and the pressure angle has most effect on the resulting error. The resulting error due to the radial dimension of the cam profile has relatively smaller variation. In the worst case, owing to the combined effects of various design parameters, the accuracy of the follower motion may degrade considerably.

The analysis procedure presented can also be used jointly with the error analysis caused by joint clearance. The approach can be extended to others cam mechanisms and planar linkages.

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